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PERFORMANCE AND COST ANALYSIS OF A COMMAND-GUIDED BALLISTIC MISSILE RADAR

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U.S. ARMY MISSILE RESEARCH AND DEVELOPMENT COMMAND

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This report addresses many of the technical questions associated with developing and implementing a command-guided ballistic rocket using a microwave or millimeter radar to provide position and roll information for the rocket. The basic feasibility of an X-band radar for providing the rocket position and roll data was established in the first phase of this design study.

The second phase of the investigation concentrated on providing additional depth in the treatment of the missile retroreflector configurations, more (Cont'd)

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detailed performance and cost trade-offs among the beacon and passive reflector-equipped missile concepts, missile beacon transponder design tradeoffs and a recommended design configuration, and an analytical examination of multiple rocket tracking system concepts and tracking radar requirements.

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PREFACE

This study was performed for the Technology Laboratory, U.S. Army Missile Research and Development Command and was supported by the Scientific Services Program, Delivery Order Number 1030 for the Battelle-Columbus Laboratories (Durham Operations) under U.S. Army Research Contract Number DAAG29-76-D-0100. The authors of this report are staff members of the Engineering Experiment Station, Georgia Institute of Technology, Atlanta, Georgia.

The opinions and findings described in this report are solely those of the authors acting as independent, private consultants, and are not to be construed as representing either an official Georgia Institute of Technology or Department of Army position, unless so designated by other authorized documents.

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I. INTRODUCTION AND BACKGROUND

The U.S. Army continues to develop essentially free-flight or ballistic rockets for providing general support, counterbattery, and area fire. Two competing, free-flight, general support rocket systems (GSRS) are presently under development.[1] The GSRS, or free-flight rocket, can deliver high volumes of TOT fire for the destruction of high-value, time sensitive, area-type targets. GSRS is envisioned as a highly mobile weapon designed to complement standard field artillery and to lay down quickly a heavy load of munitions on enemy forces attacking in a "surge" scenario; i.e., a surprise attack across Europe with massed armor.

If a relatively inexpensive guidance and control package could be integrated into the free-flight rocket, such that the accuracy of these missiles was increased to the 2.0-2.5 mil range, their effectiveness against hard point targets could be significantly increased. Such a high-accuracy, command-guided ballistic rocket could replace some general and direct support cannon and artillery units in fire support roles calling for munitions to be delivered on targets with high precision and destructiveness and roles where augmentation fires are delivered in close support of maneuver elements.

An in-house R&D program at MIRADCOM resulted in a concept for a relatively low-cost, command-guided ballistic rocket with projected accuracy significantly better than the best accuracies obtainable with the completely free-flight rocket.[2] In this concept, the missile is equipped with a receiver, a low-cost guidance package, a unique polarization sensitive passive retroreflector to provide roll position information, and side thrusters to allow the ballistic trajectory to be corrected to agree with a computer-generated reference trajectory designed to produce rocket impact at the target location. In the general concept, a ground-based laser radar was employed to determine rocket spatial position and roll orientation as a function of time. The position and roll data are processed through a ground-based computer which predicts the rocket trajectory and impact point, compares this with a reference trajectory designed to impact at the target location, and then generates trajectory correction signals which are coded on the laser radar beam, or an auxilliary rf link, and transmitted back to the missile receiver and guidance control units. In this manner, the rocket's trajectory is continuously compared to a reference trajectory and trajectory corrections are made to force the rocket's predicted impact point to agree with the target location thereby, greatly increasing the impact point accuracy. This system is called GROWLAR for Guided ROcket With LAser Radar.

The primary limitation of the GROWLAR concept is weather, smoke, exhaust plume, and cloud attenuation of the laser radar signal. To circumvent this problem, a more conventional microwave or millimeter wavelength radar has been proposed to replace the laser tracker. In this case, the radar would perform the same function as the laser tracker previously described: provide position and roll data as a function of time on the missile over the trajectory flight path. The missile retroreflector might take the form of a passive Van Atta array of antenna elements which would be located in the trailing edge of one of the missile fins, or an active microwave transponder could be used to augment the missile response. The missile antenna would linearly polarize the returned, retransmitted or reflected signal so that roll orientation processing can be accomplished in exactly the same manner as with the laser tracker.

A preliminary design study of millimeter/microwave radar concepts for satisfying the GROWLAR missile tracking and command guidance requirements has been completed.[3] The primary objective of that study was to determine the feasibility of a microwave or millimeter wavelength radar for satisfying the tracker requirements in the command-guided ballistic missile concept and to develop a preliminary design for such a radar if it proves feasible. Some of the specific tasks which were a part of that study included: examination of the propagation effects on polarization of electromagnetic signals so that the various factors which produce depolarization can be identified and the accuracy of roll position estimation can be determined; analysis of various retroreflector configurations and their polarization and signature properties in comparison with the radar signature properties of the unaugmented rocket; development of several tracking radar candidates and tradeoff analysis of these systems based on such factors as range performance, clutter rejection, tracking accuracy, and complexity; and, finally, recommendation of suitable radar configuration or configurations.

The efforts described in this report and covered by Task Order Number 1030, under U.S. Army Research Contract Number DAAG79-76-D-0100, were intended to expound on, and more fully develop, these previous preliminary design tradeoff studies, particularly in the areas of multiple target tracking techniques for the missile tracking radar; transponder system analysis, design, and costing; retroreflector techniques, configurations, performance, location, and design; radar system cost tradeoffs and complete system cost projections and comparisons.

In the following sections of this report, several multiple target tracking radar concepts will be identified, analyzed, and their suitability for the GROWLAR application will be evaluated. Design information for a suitable rocket transponder will be developed and the impact on previously identified radar system parameters will be calculated. Complete system costs for both a passive reflector and active transponder-equipped rocket system concepts, including radar concepts, will be presented along with a discussion of some possible techniques for passive augmentation of the rocket radar reflectivity. Major conclusions and recommendations are presented in the last section of the report.

II. MULTIPLE TARGET TRACKING CONCEPTS

The system analyzed in the earlier study[3] considered the case for which only a single missile was tracked by the radar system. In practice, salvo firing of missiles at small intervals is desirable, with five missiles per salvo and one second intervals between missile launches being realistic values. While a separate radar could be used to track each missile, the cost and complexity of such an approach would appear prohibitive.

There are a number of concepts which may be used to track multiple targets in addition to having a number of radars equal to the number of targets to be tracked. These concepts include:

- Off-Axis Monopulse Tracking
- Mechanically Scanned Monopulse
- Track-While-Scan (TWS) Radar
- Combined Monopulse and TWS
- Beam Agile, Phased Array Monopulse

The use of off-axis monopulse tracking is permitted by the relatively small dispersion, typically ± 30 mils about the mean trajectory[4], which would normally be expected when viewing a salvo of GROWLAR missiles. The use of track-while-scan techniques for multiple target tracking, using electromechanical scanning, frequency scanning, or phase scanning, is quite applicable to this case, and the small sector to be scanned may reduce the cost of the scanning antenna significantly. Combinations of TWS and monopulse technology (e.g. using monopulse in azimuth and TWS for elevation scanning) may provide an opportunity for a low cost, simple yet effective, system. The use of a fully beam agile monopulse (such as the PATRIOT) would satisfy requirements for multiple target track but would be rather complex and expensive; use of limited scan concepts (as employed in the TPN-19) may appreciably reduce cost of such systems.

The application of any of these concepts to the GROWLAR concept is complicated by the requirement for dual polarized operation for roll angle determination and the fact that frequency scanning is limited to those cases which do not use a beacon on board the missile. Also, coherent (MTI) operation for rejection of rain, cloud, and land clutter must not be significantly degraded. These concepts are treated in additional detail in the following sections.

A. Off-Axis Monopulse Tracking

The use of off-axis monopulse multiple target tracking offers several advantages, including the use of an antenna, transmitter, and receiver no more complex than—and, in fact, identical to—the dual polarized single target tracking system described earlier.[3] There are two distinct cases to consider: the radar case (passive reflector) and the beacon or transponder augmented case. Due to the attractiveness of this concept, it will be analyzed in considerable detail in the following sections.

1. Radar (Retroreflector) Case

The fact that a monopulse radar forms the ratio of the difference signal to the sum signal, and that this ratio is a measure of the displacement of the target from the axis, may be used for tracking of multiple GROWLAR missiles. This multiple target tracking would be simplified by the fact that the missiles would always be separated in range. The performance of a monopulse radar tracking target displaced from the null axis has been analyzed[5] and results indicate that accuracy decreases with increasing displacement from the axis. This loss of track accuracy due to off-axis tracking may be reduced by making the ratio of dispersion to beamwidth smaller; i.e., for fixed dispersion, increase the beamwidth. However, increasing the beamwidth reduces antenna gain, thus increasing thermal noise errors.

If the dispersion is fixed, there is a value of antenna beamwidth which minimizes the overall tracking error. This value of optimum beamwidth may be derived by taking a representative beamwidth value and observing the change in accuracy as the beamwidth is changed.

In order to simplify the analysis, a Gaussian beamshape was assumed. It has been shown that such an approximation is quite accurate to the -10 dB point for a variety of antenna beamshapes[6]; results were not significantly different from sin x/x patterns. Thus, if β is the 3 dB beamwidth, then the variation in voltage gain of the antenna is given as

$$S = e^{-1.388\theta^2/\beta^2} \tag{1}$$

Sharenson[5] has shown that the off-axis track error is given by

$$\sigma_T = \frac{\beta \left(1 + \frac{\theta^2}{\beta^2} k_m^2\right)^{\frac{1}{2}}}{k_m (S/N)\, S^2(\theta)} , \tag{2}$$

for the case of uncorrelated noise in the sum and difference channel, where

σ_T is the rms tracking error in degrees;

β is the 3 dB beamwidth of the antenna in degrees;

k_m is the error slope of the antenna (typical 1.6/beamwidth);

$S(\theta)$ is the reduction is voltage gain of the antenna at angle θ;

and S/N is the on-axis signal-to-noise voltage ratio.

Values of the increase in rms track error as the value of θ/β is increased are given in Table I for sin x/x and Gaussian patterns.

Thus, proper choice of beamwidth, or the ratio of beamwidth to error, can strongly impact the achievable tracking error.

If one takes as a reference system a 1° (17 mil) Gaussian beamwidth system, and multiples the beamwidth by a factor α, observing that S/N varies as $1/\alpha^2$, the new error σ_T' is given by

$$\sigma_T' = \frac{\beta}{S/N\, k_m} \left(1 + \frac{\theta^2 k^2}{\beta^2 \alpha^2}\right)^{\frac{1}{2}} \alpha^3 e^{2.78 \frac{\theta^2}{\beta^2} \frac{1}{\alpha^2}} , \tag{3}$$

or the on-axis track error $\beta / S/N\, k_m$ was modified by a factor $\gamma(\alpha)$, where

$$\gamma(\alpha) = \left(1 + \frac{\theta^2 k^2}{\beta^2 \alpha^2}\right)^{\frac{1}{2}} \alpha^3 e^{2.78 \frac{\theta^2}{\beta^2 \alpha^2}} , \tag{4}$$

TABLE I

INCREASE IN TRACKING ERROR AS TARGET MOVES OFF BORESIGHT
FOR GAUSSIAN AND SIN X/X ANTENNA PATTERNS

Angle in Beamwidths θ/β	Increase in Track Error-Gaussian	Increase in Error-sin x/x
0.0	1.0	1.0
0.1	1.05	1.05
0.2	1.22	1.22
0.3	1.59	1.55
0.4	2.20	2.15
0.5	3.25	3.25
0.6	5.16	7.68
0.7	8.67	8.33
0.8	15.70	16.49
0.9	28.22	15.36

or for $\theta = 30$ mils $\beta = 17$ mils

$$\gamma = \left(1 + \frac{7.97}{\alpha^2}\right)^{\frac{1}{2}} \alpha^3 \ e^{\frac{8.66}{\alpha^2}} \tag{5}$$

Figure 1 presents a plot of γ vs α for parameters assumed earlier, indicating a minimum value of γ of approximately 93.32 occurring for a value of α of approximately 2.65. The corresponding beamwidth is approximately 2.65 degrees, but track errors have become quite large, at a range of 30 km approaching ten mils. However, use of transponder augmentation permits a substantial reduction of these tracking errors.

2. Beacon/Transponder Augmented Case

The case for a missile augmented with an active transponder offers the potential for substantially improving performance over the passive reflector case. There are two reasons for this; the first being the higher signal-to-noise ratios achievable with the augmented system, and the second is that the error expressions indicate a less dramatic increase in track error with increases in off-axis angle.

The off-axis monopulse error for the beacon-transponder case was not derived by Sharenson[5], but a relatively straightforward extension of his analysis yields the desired results.

Using Sharenson's expression for the standard deviation of the ratio of the difference signal to sum signal, σ_V, for noise uncorrelated between the sum and difference channels (this noise has standard deviation σ),

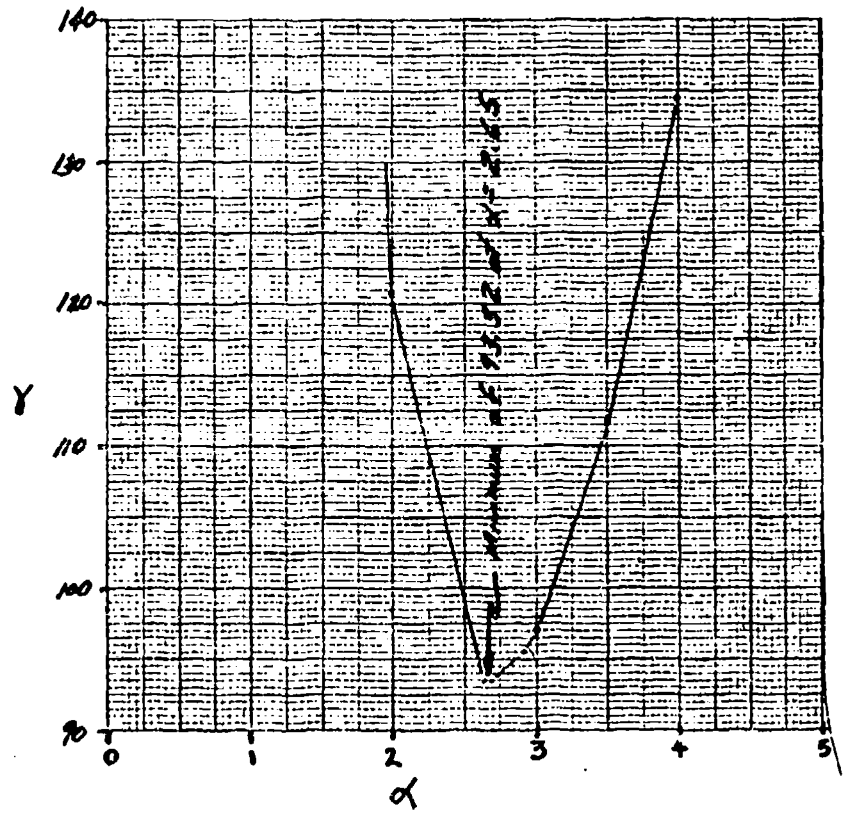


Figure 1. γ vs α for the Two-Way (Passive Reflector) Case

$$\sigma_v = \left[\frac{\sigma^2}{\Sigma^2} + \frac{\Delta^2 \sigma^2}{\Sigma^4}\right]^{\frac{1}{2}} . \tag{6}$$

where

Δ is the difference signal given by $KD(\theta)$ where K is a constant and $D(\theta)$ is the change in difference pattern with angle.

Σ is the sum signal given by $KS(\theta)$, where $S(\theta)$ is the change of sum pattern with angle.

One may then write

$$\sigma_v = \left[\frac{\sigma^2}{K^2 S^2(\theta)} + \frac{K^2 D^2(\theta)\ \sigma^2}{K^4 S^4(\theta)}\right]^{\frac{1}{2}} \tag{7}$$

$$= \frac{\sigma}{S(\theta)K} \left[1 + \frac{D^2(\theta)}{S^2(\theta)}\right]^{\frac{1}{2}} . \tag{8}$$

Note that

$$\frac{\sigma}{K} = N/S$$

$$\sigma_T = \frac{\sigma_v}{K_m}$$

$$k_m \theta = \frac{D(\theta)}{S(\theta)} ,$$

then

$$\sigma_T = \frac{\beta}{(S/N)k_m\, S(\theta)} \left[1 + k_m^2\, \theta^2\right]^{\frac{1}{2}} \quad . \tag{9}$$

This is an expression similar to the retroreflector case but with a factor of $S(\theta)$ rather than $S^2(\theta)$ in the denominator. This is intuitively satisfying since this reflects the one-way rather than two-way effects of the sum pattern antenna gain variation.

As before, if β is multiplied by a constant α, we can note the change in error γ as given by

$$\gamma = \left(1 + \frac{7.97}{\alpha^2}\right)^{\frac{1}{2}} \alpha^2\, e^{4.33/\alpha^2} \quad , \tag{10}$$

noting that S/N varies as ½ for the one-way case.

Figure 2 presents γ as a function of α, showing a minimum occurring in γ at a value of α of approximately 2.46. Thus, an optimum value of beamwidth for the monopulse tracking radar is approximately 2.46 degrees. At a range of 30 km, a system such as described in [3] would have an error dominated by fixed errors to be approximately 0.1 mil.

3. Frequency Dependence of Track Error

Additional analyses using systematic search methods have indicated that there is, in fact, a constant ratio of off-axis angle to antenna beamwidth which minimizes overall tracking error. This ratio is different

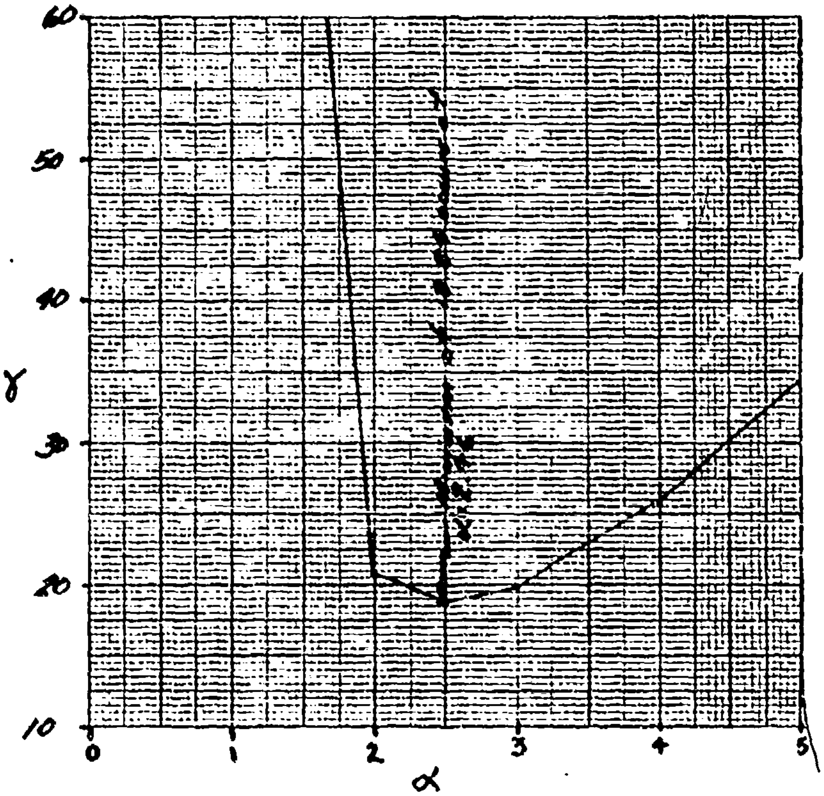


Figure 2. γ vs α for the One-Way (Transponder or Beacon) Case

for the two-way (radar) case and for the one-way (beacon transponder) case. A value of antenna beamwidth given by 1.50 times the off-axis angle maximizes tracking accuracy for the two-way case, while a value of antenna beamwidth equal to 1.39 times the off-axis angle minimizes tracking errors for the one-way case. This fact was utilized to investigate the frequency dependence of both one-way and two-way tracking system accuracies.

A set of system parameters the same as those chosen for the earlier frequency trade-off was selected for the off-axis case, except values of antenna beamwidth were chosen to be either the limiting (maximum aperture size) case, or the optimum value defined by the ratios developed above. A representative set of baseline systems is described in Table II for the passive reflector case and in Table III for the beacon-augmented missile. Since off-axis rather than on-axis null tracking is assumed in this analysis, more complete treatments of the pulse integration and track smoothing were required.

The estimate of missile position is made by the radar sensor based on the information obtained by averaging a number of received pulses together. In general, the number of pulses averaged will be close to the optimum number for range tracking which was developed in the earlier report[3]; a value of seven was chosen for this analysis. For the GROWLAR case one would expect to obtain one such measurement near the point of alignment of the received polarization with the polarization used for missile tracking. That is, one would obtain four measurements of missile position per missile revolution.

The rms tracking error due to thermal noise was calculated using procedures described in the earlier report, with a fixed instrumentation error of 0.08 mil assumed.

TABLE II

PARAMETERS FOR BASELINE SYSTEMS
OPTIMIZED FOR OFF-AXIS PASSIVE REFLECTOR TRACKING
(See Text For Details)

Configuration Number	Transmitted Power (kw)	Frequency (GHz)	Antenna Gain (dB)	Losses (dB)	Beamwidth (mils)	Off-Axis Angle (mils)	Noise. Power (dBm)	Number of Pulses Integrated	Error Slope	RCS (dBsm)	Fixed Instrumentation Error (mils)	Number of Measurements For Smoothing
1	1000	5.5	36.8*	6	42.5	5	-99	7	1.57	-7.02	0.08	36
2	1000	5.5	36.8*	6	42.5	10	-99	7	1.57	-7.02	0.08	36
3	1000	5.5	36.8*	6	42.5	20	-99	7	1.57	-7.02	0.08	36
4	1000	5.5	36.8*	6	42.5	30	-99	7	1.57	-7.02	0.08	36
5	250	9.5	41.5*	8	24.7	5	-97	7	1.57	-2.28	0.08	36
6	250	9.5	41.5*	8	24.7	10	-97	7	1.57	-2.28	0.08	36
7	250	9.5	39.8	8	30	20	-97	7	1.57	-2.28	0.08	36
8	250	9.5	36.3	8	45	30	-97	7	1.57	-2.28	0.08	36
9	125	16.5	46.3*	11	14.28	5	-96	7	1.57	2.52	0.08	36
10	125	16.5	45.6	11	15	10	-96	7	1.57	2.52	0.08	36
11	125	16.5	39.5	11	30	20	-96	7	1.57	2.52	0.08	36
12	125	16.5	35.3	11	45	30	-96	7	1.57	2.52	0.08	36

*Values Limited by 5ft Aperture Constraint

TABLE III

PARAMETERS FOR BASELINE SYSTEM

OPTIMIZED FOR OFF-AXIS BEACON TRANSPONDER TRACKING

(See Text for Details)

Configuration Number	Peak Beacon Transmitted Power	Losses (dB)	Frequency (GHz)	Fixed Instrumentation Error (mils)	Tracking Antenna Gain (dB)	Missile Antenna Gain (dB)	Tracking Beamwidth (mils)	Off-Axis Angle (mils)	Noise Power (dBm)	Number of Pulses Integrated	Number of Measurements For Smoothing
1	100 w	6	5.5	0.08	36.8	14.5	42.5*	5	-99	4	36
2	100 w	6	5.5	0.08	36.8	14.5	42.5*	10	-99	4	36
3	100 w	6	5.5	0.08	36.8	14.5	42.5*	20	-99	4	36
4	100 w	6	5.5	0.08	36.8	14.5	42.5*	30	-99	4	36
5	10 w	8	9.5	0.08	41.5	19.24	24.65*	5	-97	4	36
6	10 w	8	9.5	0.08	41.5	19.24	24.65*	10	-97	4	36
7	10 w	8	9.5	0.08	40.5	19.24	27.8	20	-97	4	36
8	10 w	8	9.5	0.08	36.98	19.24	41.70	30	-97	4	36
9	1 w	11	16.5	0.08	46.3	24.03	14.28*	5	-96	4	36
10	1 w	11	16.5	0.08	46.3	24.03	14.28*	10	-96	4	36
11	1 w	11	16.5	0.08	40.5	24.03	27.8	20	-96	4	36
12	1 w	11	16.5	0.08	36.98	24.03	41.70	30	-96	4	36

*Values Limited by Maximum 5ft Aperture Size

Estimates of missile position produced by the radar system may be further improved by smoothing or curve fitting these points. The improvement in accuracy is dependent on the exact signal processing routine utilized, and results from several analyses are briefly summarized in the following paragraphs; more complex filters requiring computer simulation[7] were not analyzed, but their performance is expected to be quite similar to those which are amenable to cloud form analysis.

Morrison[8,9] has analyzed the case for both fixed memory polynomial curve fitting and expanding memory-type filters. The results for the reduction in error variance for a fixed-memory filter are given by

$$\frac{2(2N+1)}{(N+2)(N+1)}$$

for a first degree fit, where N is the number of measurements used to perform the curve fit.

Benedict and Bordner[10] have analyzed the case for the reduction of measurement errors associated with use of the α-β filter. The variance reduction ratio associated with this filter is

$$\frac{2\alpha^2+\beta(2-3\alpha)}{\alpha[4-\beta-2\alpha]} .$$

In addition, they showed that an optimum value of β is given by

$$\beta = \alpha^2/(2-\alpha) ,$$

thus reducing the choice of parameter to the choice of a single, smoothing coefficient, α.

Quigley, *et al* [11], have analyzed the case for a Kalman filter and determined that for N measurements, the variance has been reduced by a factor of

$$\frac{2(2N-1)}{N(N+1)} .$$

Each of these analyses yields similar results (assuming α is chosen so as to provide a smoothing interval of N samples), so that the particular smoothing procedure chosen does not significantly impact predicted tracking errors. For the analysis, a reduction in variance of

$$\frac{2(2N+1)}{(N+2)(N+1)}$$

was selected.

The value of N selected thus impacts the achievable accuracy of missile location. One would want only to process data between missile side-thruster firings, which would normally preclude smoothing for more than one to three seconds. At a missile roll rate of 3 Hz, a smoothing interval of three seconds, and four measurements per revolution, a value of N = 36 is obtained, and this value was used in the accuracy analysis. Following the same procedures which were set forth in the earlier analyses, thermal noise and instrumentation errors were combined in a root-mean-square fashion. Some representative results are summarized in Figures 3 through 7. Boundaries of the radar track accuracy region are set by the on-axis tracking error (the minimum error), and the track error for a target at the edge of the dispersion region. Intermediate values of dispersion will fall between these two extremes.

Figure 3 shows tracking error as a function of range for the 5.5 GHz tracking radar described in Table II. Values of on-axis tracking, 20 mil off-axis tracking, and 30 mil off-axis tracking are given on the same graph because the constant aperture limit resulted in a non-optimum aperture-limited beamwidth for this particular condition.

Analysis at a frequency of 9.5 GHz is given as Figure 4, showing the track accuracy region for a 10 mil dispersion of missiles, bounded by the on-axis and the 10 mil off-axis tracking errors. Figure 5 shows the same presentation for a 9.5 GHz system, but optimized for 30 mil off-axis tracking.

Figures 6 and 7 give similar results for 10 mil dispersion and 30 mil dispersion respectively for an operating frequency of 16.5 GHz.

Each of these analyses indicates substantial tracking errors associated with the requirement for off-axis tracking. It should particularly be noted that maximum track accuracies are a strong function of the required dispersion associated with the missile rounds to be tracked. In fact, errors exceed the acceptable minimums in every case within the 40 kilometer maximum track range.

Examination of the data presented for the tracking of a passive reflector show excessive errors at longer ranges. In order to reduce these errors, the performance of a similar system but with 10 dB of pulse compression incorporated (this would give a duty cycle of 0.01, a practical upper limit for available transmitter tubes) was examined. Results of this analysis are summarized in Figure 8, illustrating improvement in accuracy, but still evidencing substantial errors at the longer ranges.

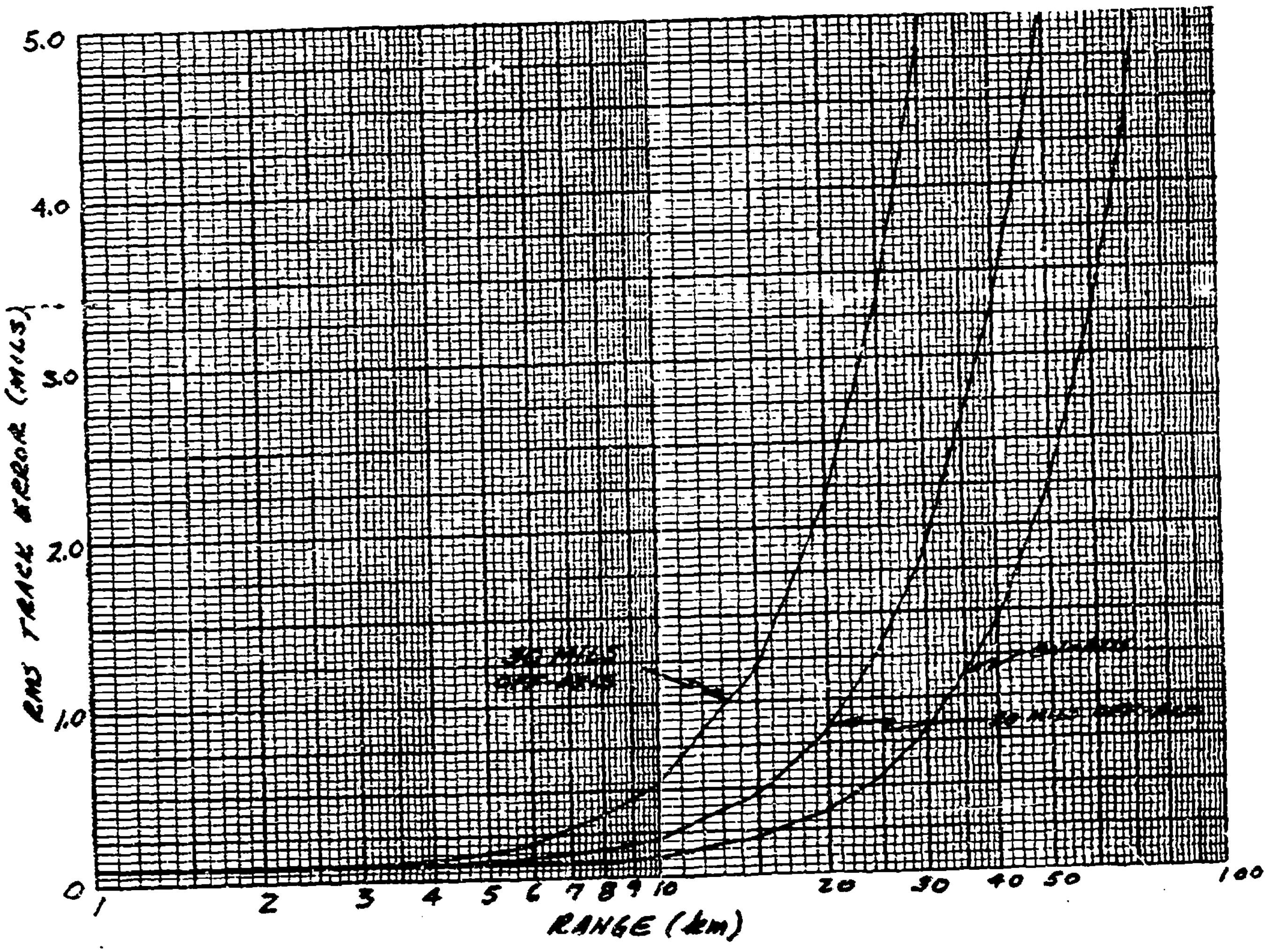


Figure 3. Tracking Error as a Function of Range for the 5.5 GHz System (Configuration 1-4) Described in Table I. A Passive Reflector Was Used, and the 5ft. Aperture Limit Was the Governing Factor So Beamwidth Was Not Optimized

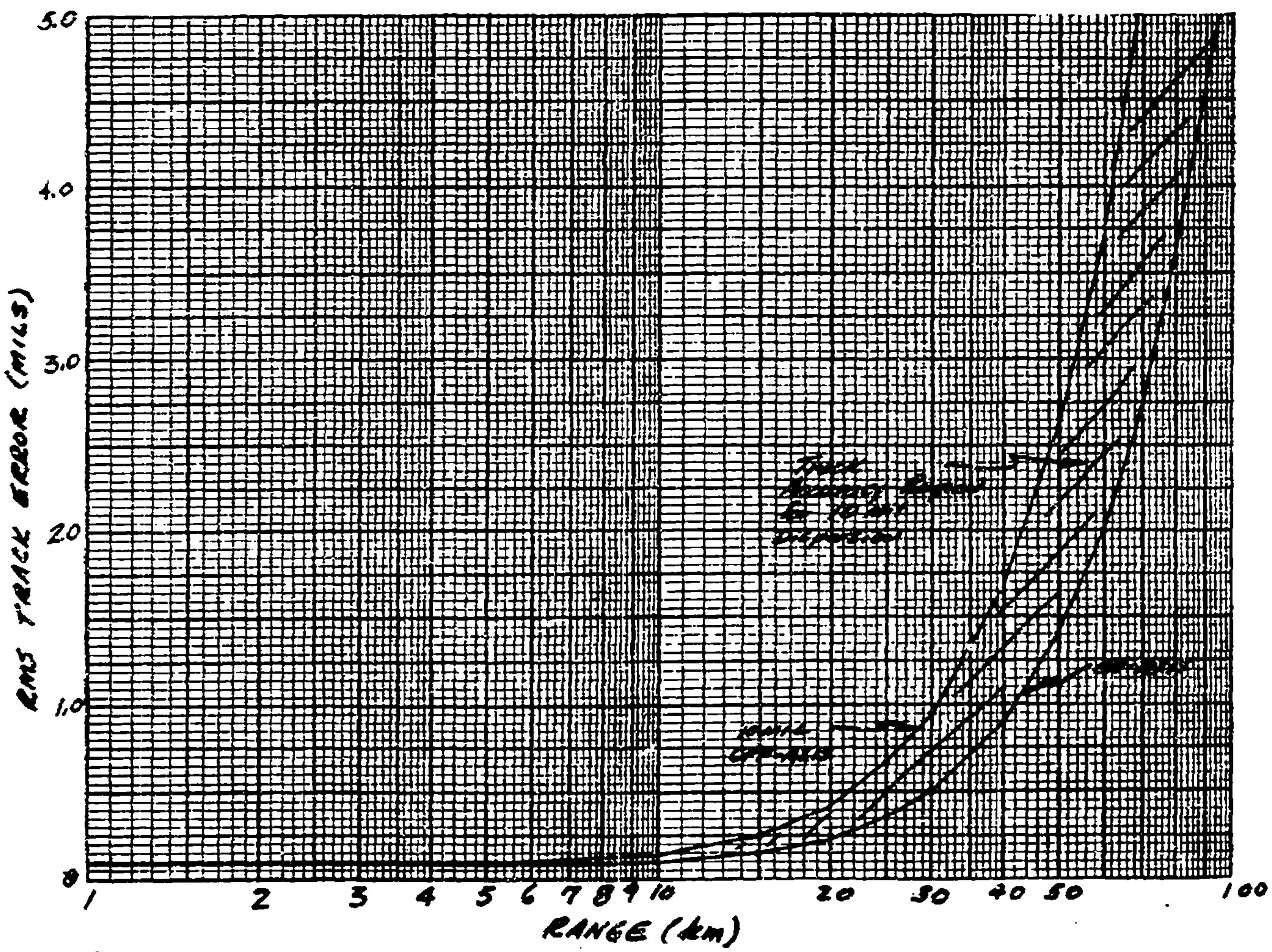


Figure 4. Tracking Error for Radar Configuration 6 of Table I, Tracking a Missile with a Passive Reflector. Optimized for 10 mil Off-Axis Tracking at a Frequency of 9.5 GHz

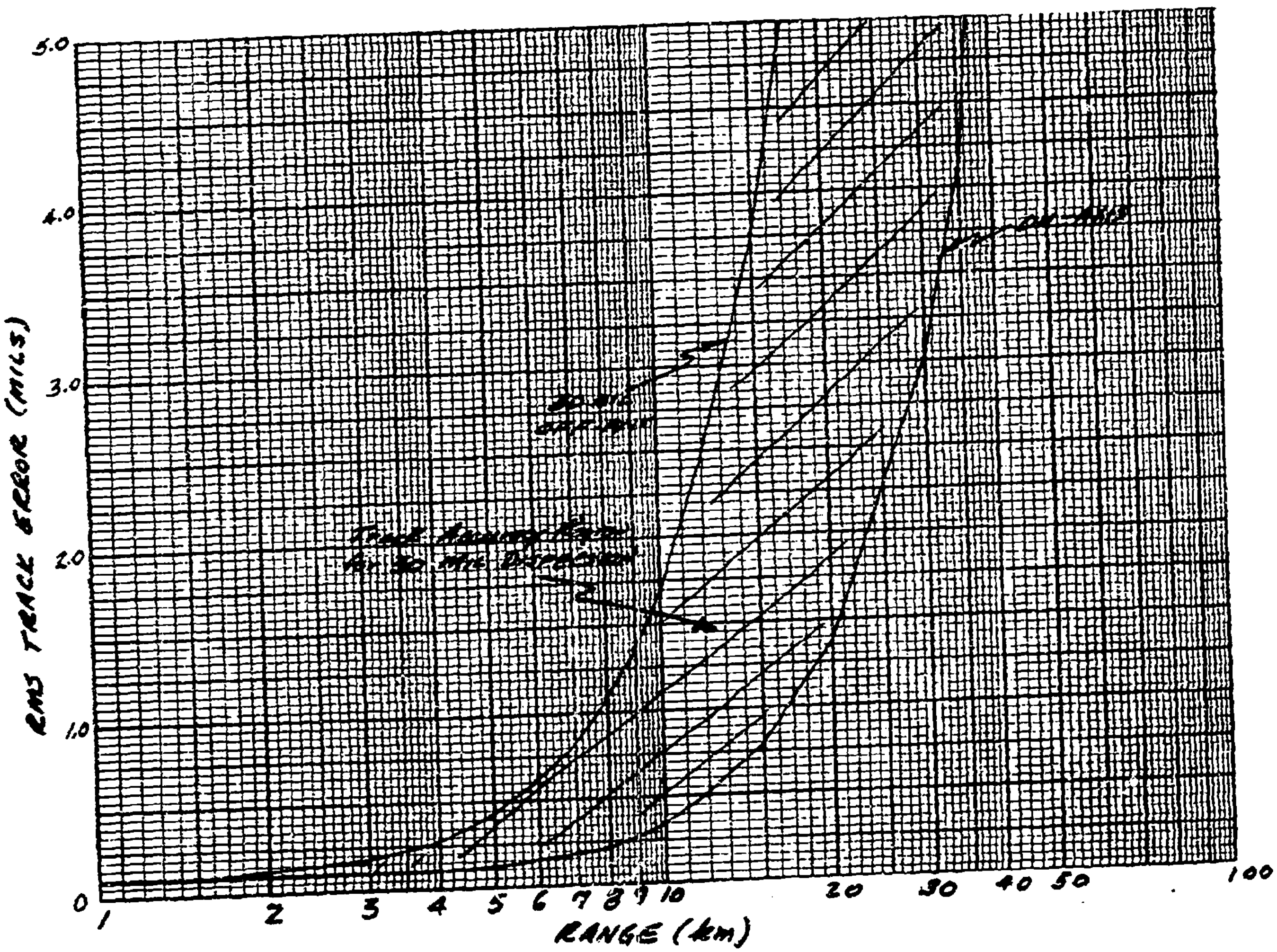


Figure 5. Tracking Error for Radar Configuration 8 of Table I, Tracking a Missile Equipped with a Passive Reflector. Optimized for 30 mil Off-Axis Tracking at 9.5 GHz

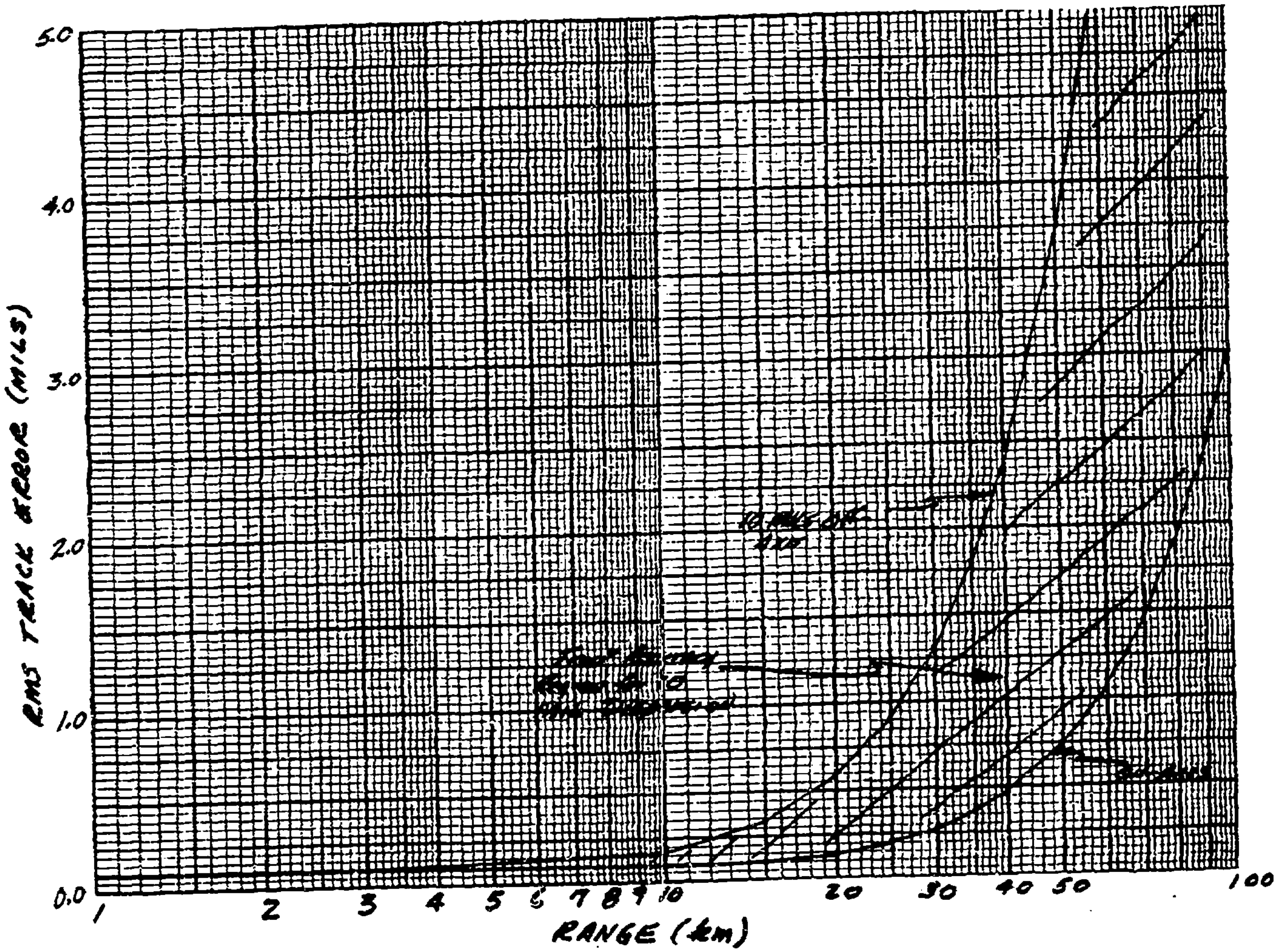


Figure 6. Tracking Error for Radar Configuration 10 of Table I, Tracking a Missile Equipped with a Passive Reflector. Optimized for 10 mil Off-Axis Tracking at 16.5 GHz

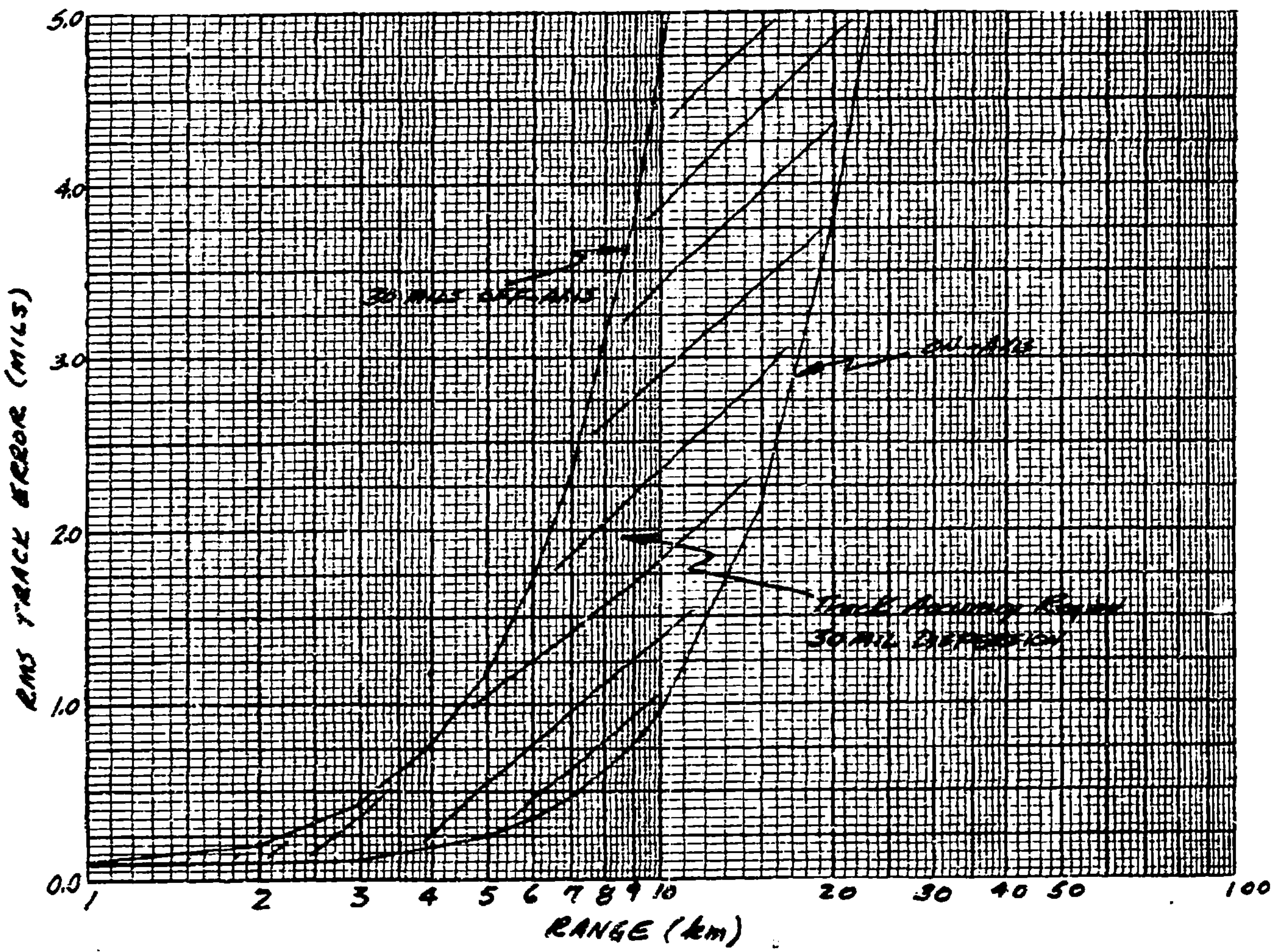


Figure 7. Tracking Accuracy for Radar Configuration 12 of Table I, Tracking a Missile Equipped with a Passive Reflector. Optimized for 30 mil Off-Axis Tracking at 16.5 GHz

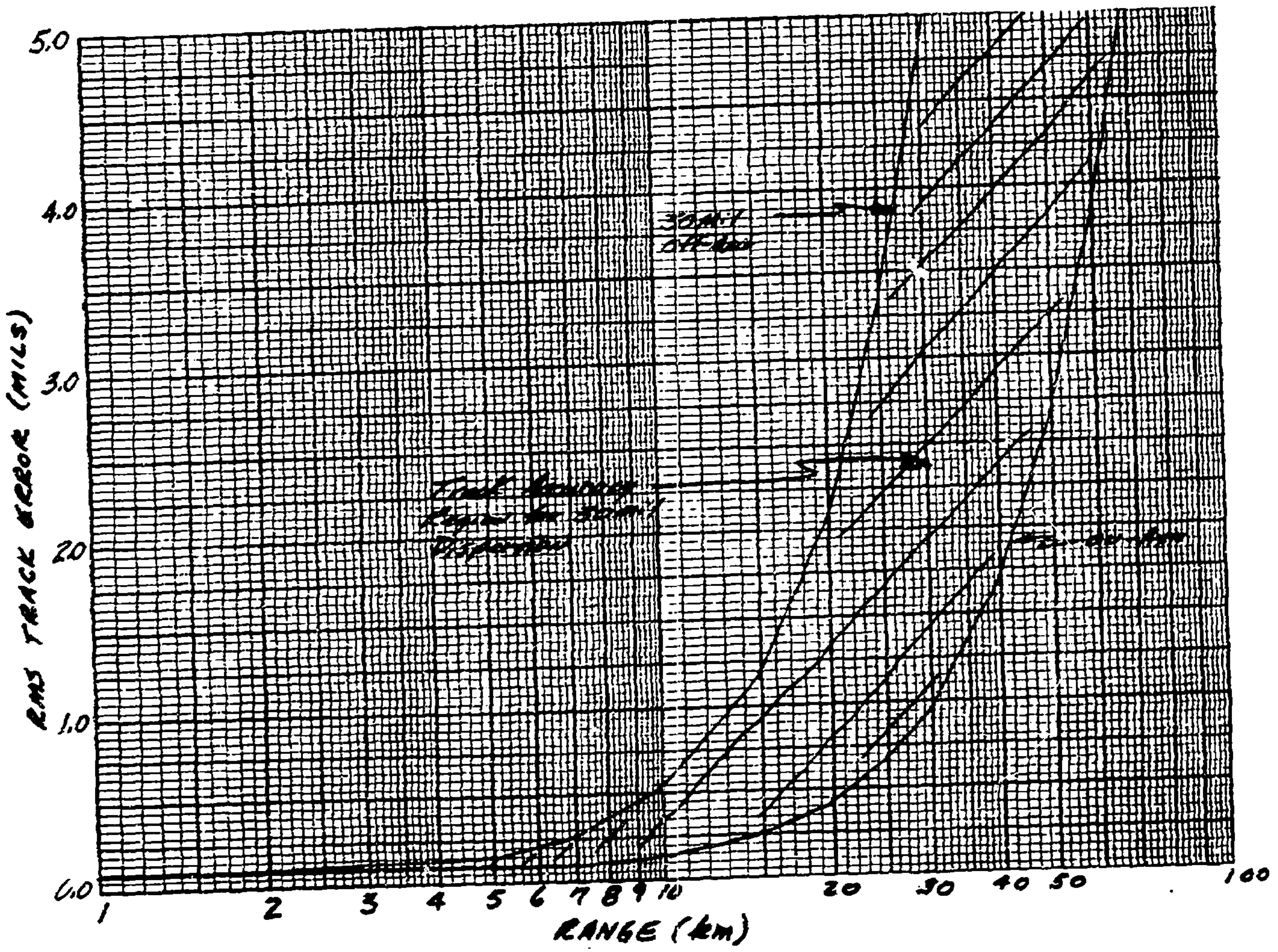


Figure 8. Tracking Error for Radar Configuration 12 of Table II but with 10 dB of Pulse Compression Gain Incorporated, Tracking a Missile Equipped with a Passive Reflector. Optimized for 30 mil Off-Axis Tracking at 9.5 GHz

When a transponder-equipped missile is tracked using off-axis monopulse tracking, quite impressive accuracies may be achieved. A representative set of such transponder-equipped systems is described in Table III, and similar beamwidth optimization and tracking error analyses were carried out. In almost all cases, the tracking error was dominated by the fixed instrumentation error, with the exception of the 20 mil and 30 mil dispersion cases for operation at 16.5 GHz, whose performance is summarized in Figures 9 and 10.

Since operation for all dispersions up to 30 mils resulted in a 0.1 mil error for the 5.5 and 9.5 GHz cases, those are not presented in graphical form. Similarly, the tracking error for 16.5 GHz operation optimized for 5 and 10 mil dispersions was dominated by the 0.08 mil instrumentation error over the range interval out to 40 kilometers, and is not plotted.

These analyses indicate for the frequencies considered, which are those which would be expected to yield reasonable performance in adverse weather, operation of an optimized off-axis tracking radar following a missile equipped only with a passive reflector results in relatively large track errors at ranges less than 40 kilometers. Operation while tracking a transponder-equipped missile results in substantial improvements in overall system accuracy, system accuracy being dominated by instrumentation errors rather than thermal noise-induced tracking errors in almost all cases analyzed.

While the transponder-equipped system will provide accurate tracking for GROWLAR missiles, a desirable system alternative is to be able to operate with a missile which is entirely passive. In order to accomplish this with a conventional monopulse system operating in the off-axis tracking mode,

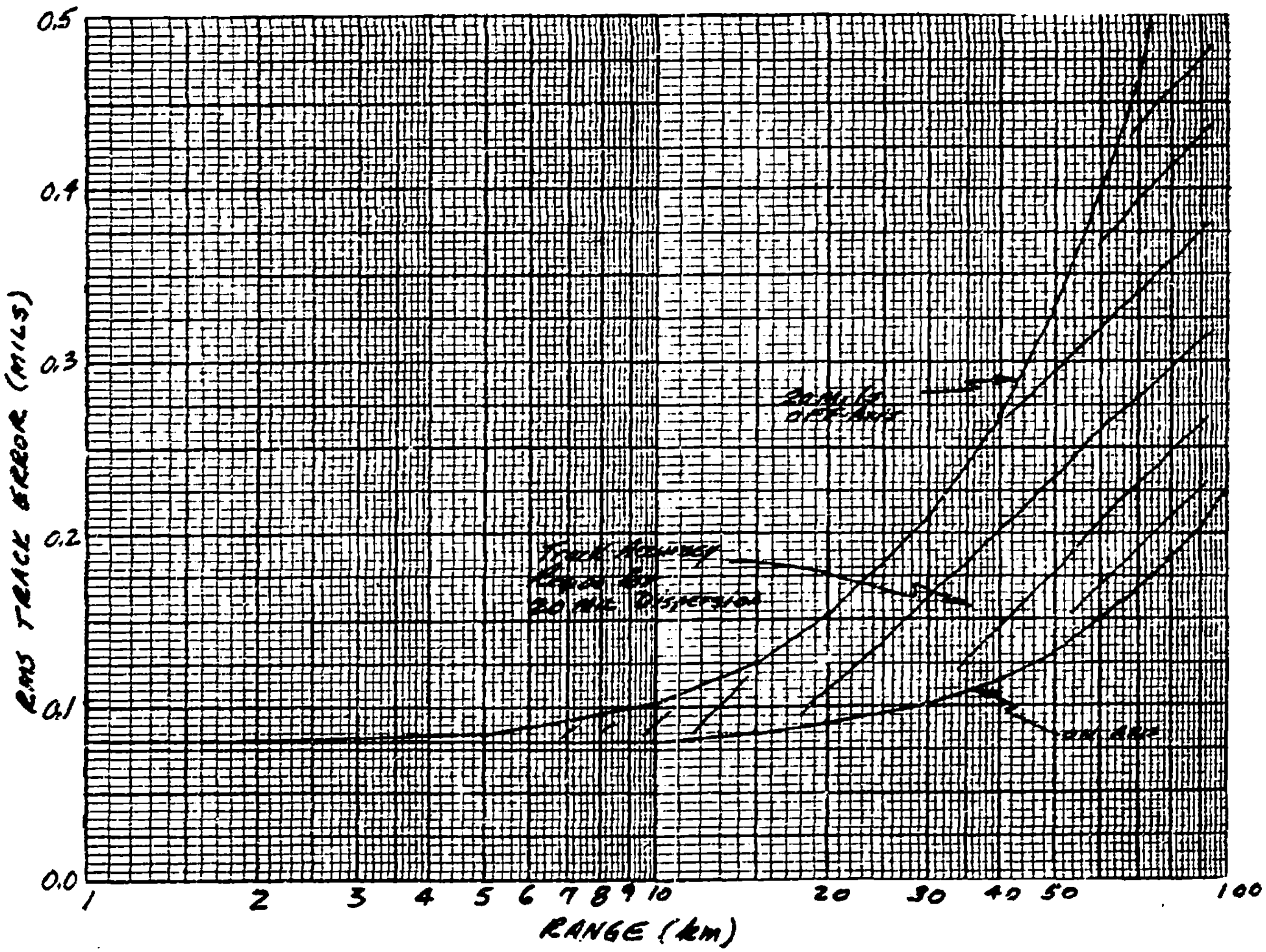


Figure 9. Tracking Error as a Function of Range for the 16.5 GHz Beacon-Transponder System Described as Configuration 11 of Table II. Parameters Optimized for 20 mil Off-Axis Tracking

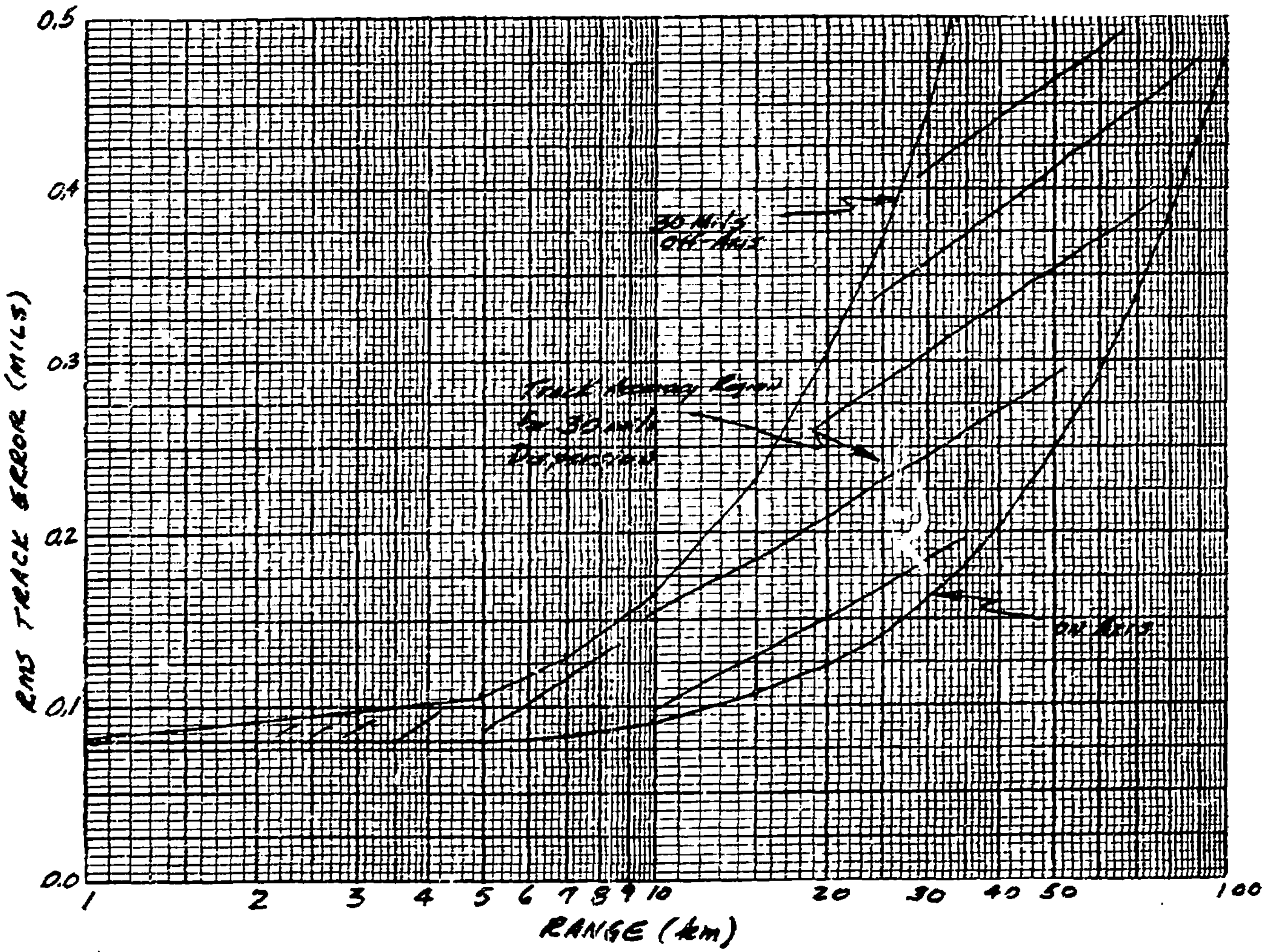


Figure 10. Tracking Error as a Function for the 16.5 GHz Configurat'on 12 of Table II. Parameters Optimized for 30 mil Off-Axis Tracking

significant increases in average power on the target, and substantial, additional signal processing complexity would be required. Thus, other tracking techniques for the dispersed missile scenario were investigated and results presented in the following pages.

B. Mechanically Scanned Monopulse Trackers

One of the significant problems with the off-axis angle track concepts discussed in the previous section is that the required off-axis angle was relatively large, resulting in a reduction in track accuracy for such targets. The use of a mechanically scanned monopulse system which can be generated by motion of the feed, sub-reflector, or primary reflector of a conventional monopulse system, can significantly reduce these off-axis angles at the time of measurement, thus increasing system tracking accuracy. However, on closer examination, there are significant difficulties encountered when attempting to apply this concept for the GROWLAR application.

The principal difficulty when using electromechanically scanned monopulse beams is that the high roll rate of the missile requires a rapid scan over the entire sector of interest in order to make accurate polarization measurements on each target and to update missile roll and track information at a reasonably high rate. Such a rapid, continuous scan results in relatively few pulses on the target when the target is located near the boresight of the antenna beam, significantly reducing the number of samples available for integration in order to minimize track errors. Rapid motion of the beam across the target also introduces additional pulse-to-pulse amplitude fluctuations which degrade the clutter cancellation of any system which employs coherent processing for elimination of returns from clouds, precipitation, or land clutter.

Another difficulty when using electromechanically scanned monopulse beams involves the careful control of the polarization properties of the system which is necessary in order to make a polarization roll angle determination. A survey of available literature uncovered no treatment of such effects but examination of co-polarized returns produced by such scanning[12,13,14] indicates that the problems may not be unsurmountable, particularly when only modest polarization accuracy is required.

The high mechanical scanning rates which are required in order to update target track and polarization information may adversely affect the reliability of the mechanical mechanism utilized to generate such scanning. However, careful design should minimize such problems.

In summary, the rapid, continuous mechanical scanning of a monopulse system over a limited angular section results in a relatively inefficient utilization of the available energy from the radar. This inevitably results in a significant degradation in angular tracking accuracy over that which would be acquired if the radar energy could be accurately placed upon each desired target with the sector of interest.

C. Track-While-Scan (TWS) Radar

It is possible to use track-while-scan (TWS) concepts to track the GROWLAR missiles. In a TWS system, a conventional single antenna beam is scanned over the sector of interest and target position is measured based on variations of the amplitude of received signal as the antenna beam is scanned across the target. Rather accurate measurements of angular position may be accomplished utilizing such an approach. Figure 11 shows the theoretical accuracy of a track-while-scan system for eight pulses per beamwidth,[15] while Figure 12 shows results obtained from a Monte Carlo simulation of a

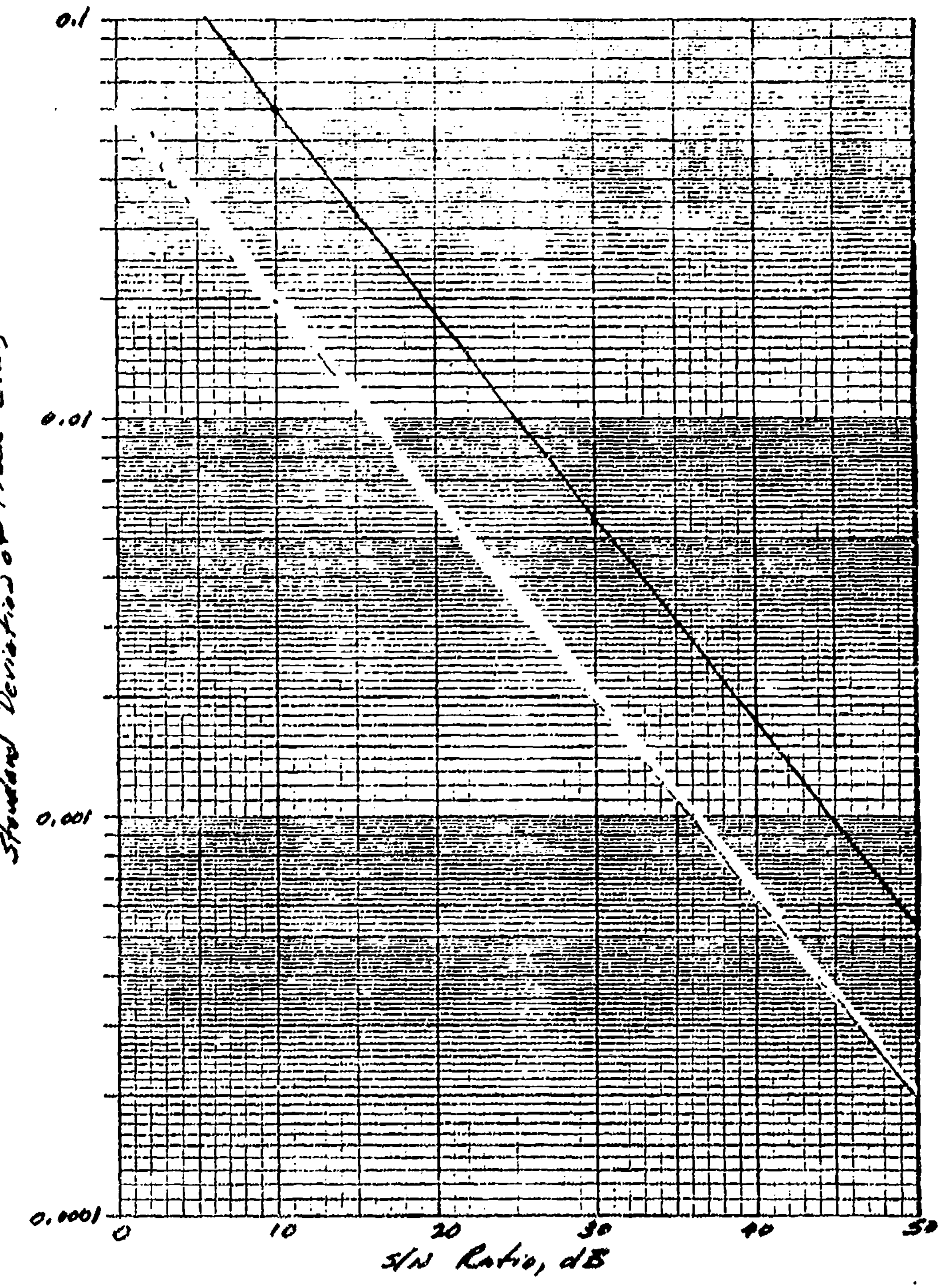


Figure 11. Relationship Between S/N Ratio and Track Error in Beamwidths for an Ideal TWS System, Eight Pulses Per Beamwidth

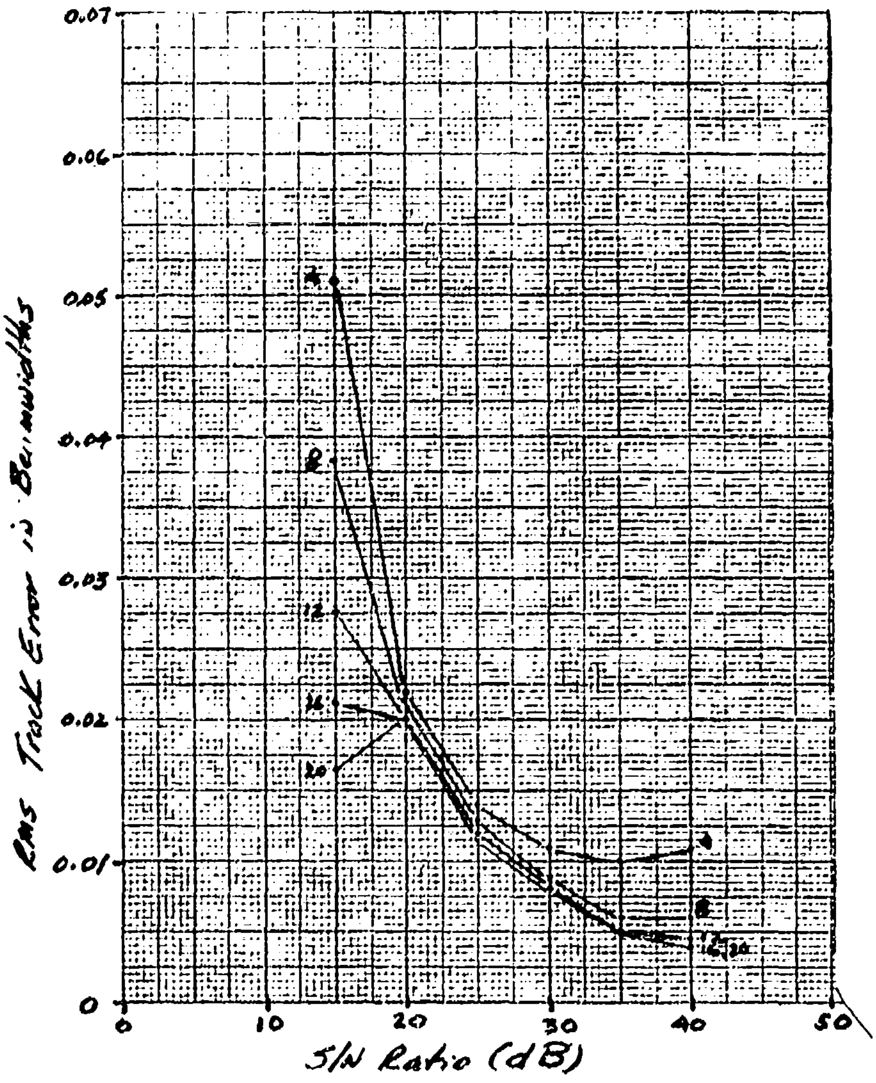


Figure 12. Track Error vs SNR for Various Numbers of Pulses per Beamwidth. Linear Fit to Derivative, 16 Bit A/D

TWS system. The Monte Carlo simulation actually introduced such factors as the specific computational algorithm which was utilized and the effects of A-to-D quantization noise on overall tracking performance. Both of these figures illustrate that a TWS system can generate accurate track information if a large number of pulses per beamwidth is achieved.

In actual implementation, usually two antenna beams are utilized in order to scan the sector of interest. The beams are normally narrow in one dimension but sufficiently broad in their orthogonal dimension as to cover the desired scan sector. This broadening of the beam reduces antenna gain over that of a similar pencil beam. The requirements for scanning of two beams requires either the utilization of two separate transmitters or the time sharing of a single transmitter between the two beams. Thus, there is a significant reduction in energy per beam on the target.

Scanning of the TWS antennas may be accomplished by electromechanical scanning, frequency scanning, or phase scanning as in a phased array system. The cost of phase scanning can be quite appreciable and frequency scanning can, in general, only be accomplished in a single plane--at the expense of increased transmitter complexity. Use of electromechanical scanning antennas requires the additional complexity associated with two devices such as a Foster scanner, a Lewis scanner, or a Geodesic Luneburg lens. Such devices are normally not dual polarized but the polarization of the signal from the missile must be measured with both of the scanners or antenna beams which are utilized. It is conceptually possible to provide a dual polarization capability using polarization transforming grids and additional focusing elements, but such approaches increase cost and complexity.

In order to update position and polarization information from the target, the beam must be rapidly scanned over the sector. However, this results in relatively small numbers of pulses received from any given target with a consequent reduction in tracking accuracy. These rapid fluctuations in received signal strength also reduce the clutter rejection capability of the system. Tracking accuracy is further reduced by any fluctuations in received signal strength due to rotation and wobble of the missile during flight. These combinations of factors act to impact, significantly, the achievable tracking errors.

In order to probe the effect of limited numbers of pulses per beamwidth on clutter cancellation or improvement, an analysis was carried out to determine the relationship between numbers of pulses per beamwidth and the pulse-to-pulse fluctuation levels. The analysis assumed a Gaussian antenna beamshape; fluctuation levels were calculated and translated to improvement limits for both the on-axis case and the pulses occurring at the 3 dB point of the antenna. Results of this analysis are given as Figure 13, showing the range of achievable clutter cancellation, if the limits are due to beam motion alone.

The fact that the motion of the beam across the target, whether step-scan or continuous, limits the achievable clutter cancellation to relatively small values, represents a significant problem associated with any TWS system. This improvement limit applies, regardless of the means by which such scanning is generated, and makes the use of monopulse systems, which require no such scanning across the target, most attractive.

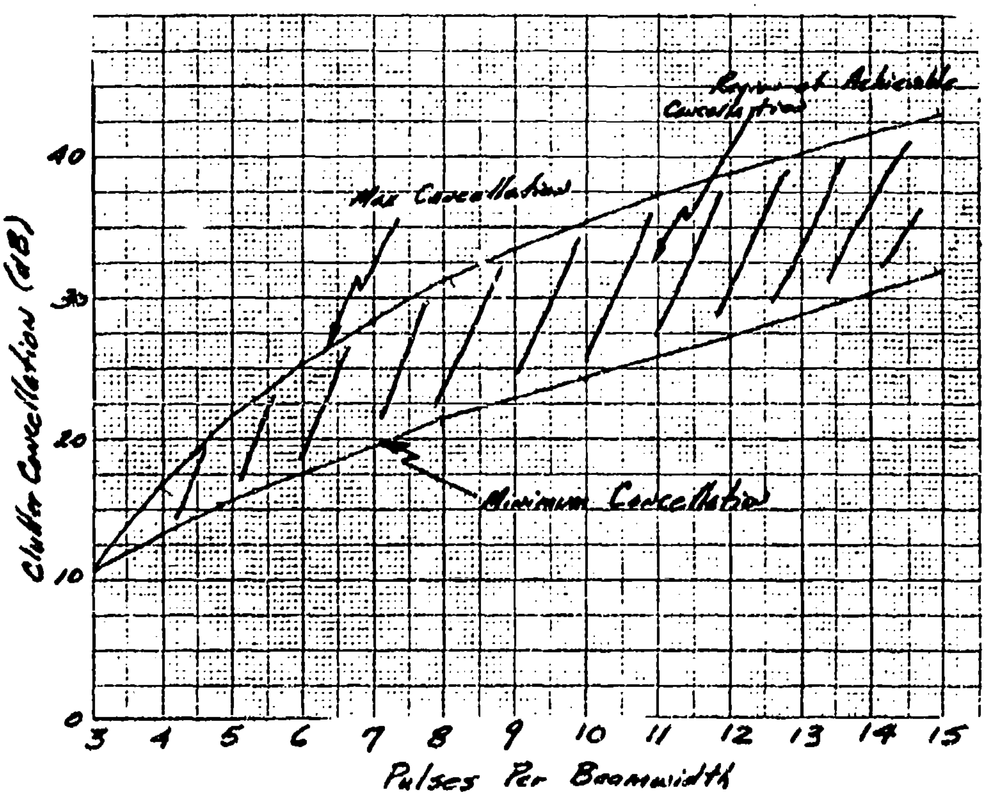


Figure 13. Bounds on Clutter Cancellation as a Function of Number of Pulses Per Beamwidth for a Gaussian Beam Pattern

Summarizing, while a TWS system is capable of accurate angle measurements, the particular requirements of the GROWLAR result in degraded accuracy and unattractive system complexity. The rapid scan rate required to update the polarization-roll angle data at a rate high compared with the roll rate of the missile results in relatively few pulses per beamwidth, an inefficient use of radiated energy and a reduction in clutter rejection capability. The system complexity for a dual polarization measurement with what are fundamentally single-polarized systems is unattractive from both a cost and a reliability viewpoint.

D. Combined Monopulse and Track-While-Scan

The narrow dispersion of the missiles in azimuth and the broad dispersion in elevation makes consideration of an antenna concept involving scanning in elevation (either frequency, phase, or electromechanical) coupled with monopulse tracking in azimuth to be quite an attractive approach. Unfortunately, the elevation tracking problem presents many of the same difficulties associated with a conventional track-while-scan system, complicated by the fact that elevation errors produce the greatest effect on predicted impact point (see Appendix).

The principal advantage is that scanning is required in a single plane only, with the consequent use of a high-gain, narrow beam antenna. However, all of the problems associated with limited numbers of pulses on the target, amplitude fluctuations, and difficulties of dual polarized measurements remain.

If tracking of a beacon is not required, frequency scan could be used for scanning the beam in the elevation coordinate. Systems using frequency scan in a single coordinate have been widely used in operational systems[16] and dual polarized frequency scanning systems suitable for partitioning to obtain a single coordinate monopulse capability have been proposed[17], but not fabricated. The utilization of frequency scanning permits true beam agility; that is, the beam may be scanned over only those regions which contain the targets of interest. However, this capability is only achievable if the transmitter has a high degree of flexibility and stability.

This concept of combined monopulse and track-while-scan is, perhaps, somewhat more attractive than a conventional track-while-scan but still evidences significant problems, including small numbers of pulses on target, clutter cancellation limitations, requirements for dual polarized operation, and stringent transmitter requirements if frequency scanning is used.

E. Beam Agile Phased Array

The use of a fully beam agile system, such as utilized in the PATRIOT radar system, provides a means of accurately apportioning energy to the various targets within the scan sector while also providing the capability for accurate tracking of these various targets. Unfortunately, the cost and complexity of such a system can be best described as astronomical.

However, since accurate tracking is only required over a limited region of space, recent developments in limited scan phased array monopulse antennas may be utilized. The geometry of such a system is shown as Figure 14[14] where a small phased array is used to illuminate a sub-reflector feeding an offset paraboloidal antenna, permitting beam scanning over a limited segment of space, utilizing substantially fewer phase shifters than would be required in a full-phased array system. This concept has been implemented in a somewhat simpler form in the AN/TPN-19 precision approach radar.[18,19] Figure 15

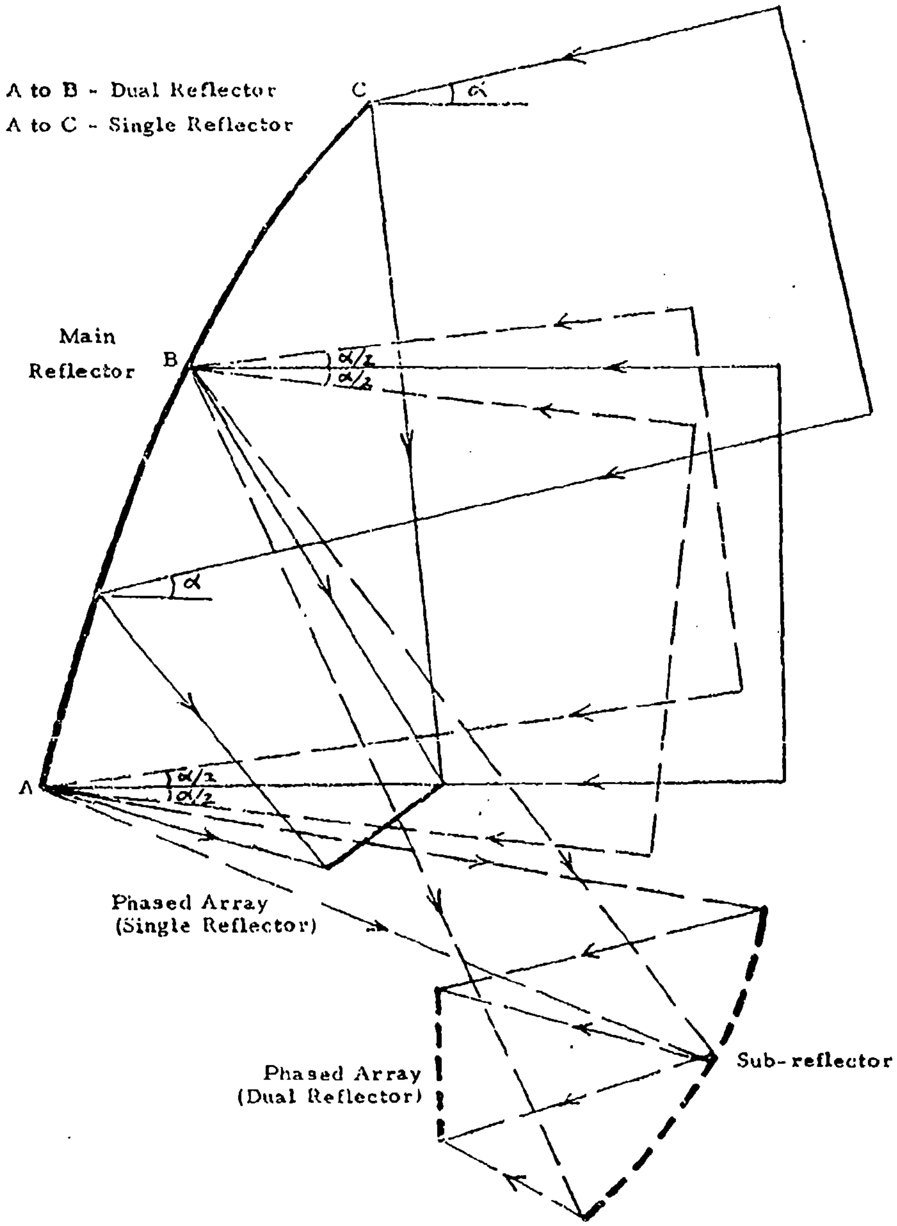
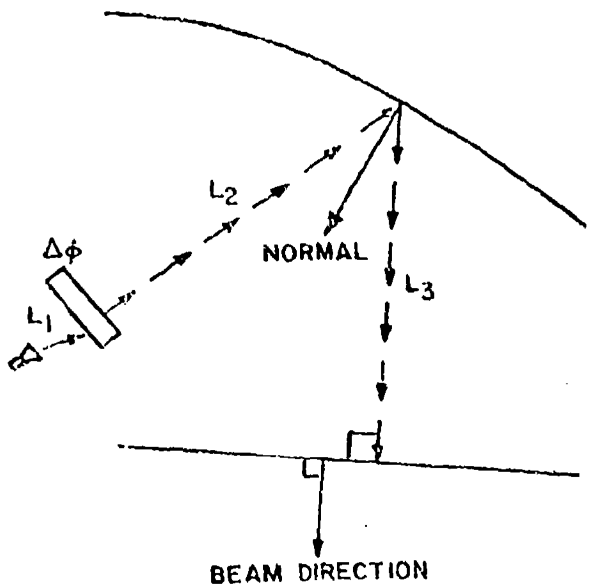


Figure 14. Diagram of a Dual Reflector Limited Scan Phased Array Antenna System [14]



Simplified Schematic View of the TPN-19 Antenna System [18]

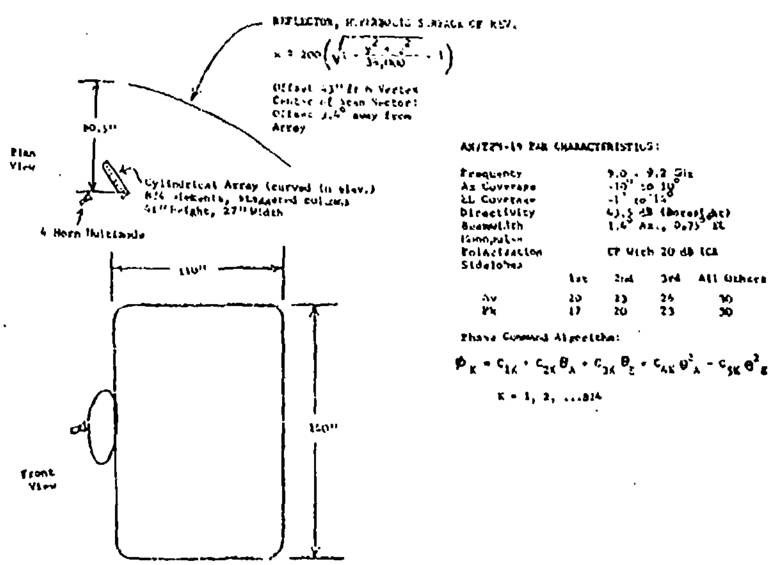


Figure 15. TPN-19 Characteristics [19]

shows a simplified view of the TPN-19 antenna system which does not utilize the sub-reflection shown in Figure 14. The characteristics of the TPN-19 system are summarized in the table also shown in Figure 15.

For the GROWLAR concept, even further simplifications may be achievable. The relatively small dispersion in elevation permits a significant reduction in number of phase shifters required, while the even smaller small dispersion in azimuth will, in all probability, permit accurate monopulse azimuth determination without motion of the beam in that coordinate.

The most significant difference between the TPN-19 and the GROWLAR application is the fact that dual polarization operation is required for the GROWLAR; the AN/TPN-19 operates with a single, circular polarization. The only difficulty in implementing the dual polarization is the fact that the phase shifters utilized in the AN/TPN-19 are circularly polarized. It appears possible to make polarization insensitive transmission phase shifters. In the event that unforeseen technical difficulties are encountered with such an approach, certainly polarization insensitive reflect array shifters[20] could be utilized on the sub-reflector of the system of Figure 14 in order to obtain limited beam scanning with a dual polarized system. Conventional dual polarized monopulse horns as described in Reference 3 could be used with either of these approaches.

In order to indicate the accuracy which may be achieved utilizing such a limited scan system, a hypothetical or baseline system described in Table IV was defined, and tracking errors determined by the methods which were outlined earlier in this section. In a beam agile phased array system, the question of smoothing of measured data becomes somewhat more complex since measurements are not continuously available from each missile. Thus, there is a reduction in

TABLE IV

CHARACTERISTICS OF BASELINE LIMITED SCAN PHASED ARRAY SYSTEM
(See Text for Details)

Parameter	Passive Reflector System	Beacon Transponder System
Frequency	9.5 GHz	9.5 GHz
Peak Power	250 kw	250 kw
Pulse Compression	10(10dB)	-
Radar Antenna Gain	41.5 dB	41.5 dB
Radar Antenna Beamwidth	25.5 mils	25.5 mils
Beam Alignment with Target	1 mil	1 mil
Reflector RCS	-2.28 dBsm	-
Missile Antenna Gain	-	19.24 dB
Beacon Peak Power	-	10 watts
Losses	8 dB	8 dB
Receiver Noise Power	-99 dBm	-99 dBm
Fixed Instrumentation Error	0.08 mils	0.08 mils
Number of Pulses Integrated	7	4
Number of Measurements Smoothed	18	18

number of measurements related to the number of missiles which are tracked by the radar. Careful beam programming may minimize effects of this multiple target tracking requirement. A value of 18 samples were used for the accuracy analysis.

For the beacon augmented case, errors were essentially instrumentation limited out to ranges in excess of 150 kilometers. For the passive reflector case, errors as a function of range were calculated and results summarized in Figure 16, maximum errors increasing to 0.25 at the 40 kilometer range.

The relatively low cost and simplicity of the limited scan concept is quite attractive for a GROWLAR application. However, the requirements for dual polarized operation would require some additional research and development but development costs should not be of such a magnitude as to be prohibitive.

F. Summary

Of the various multiple target tracking concepts investigated, the simplest of these--the use of off-axis monopulse tracking--produced satisfactory operation for a transponder augmented missile, but unacceptable errors were introduced when tracking a target with a passive reflector for dispersions of 20 to 30 mils at longer ranges.

Significant problems were encountered with mechanically scanned monopulse, TWS, combined monopulse and TWS, and fully beam agile phased array system.

The use of limited scan phased array antennas provides improved tracking accuracy for both beacons and the passive reflector case, at moderate levels

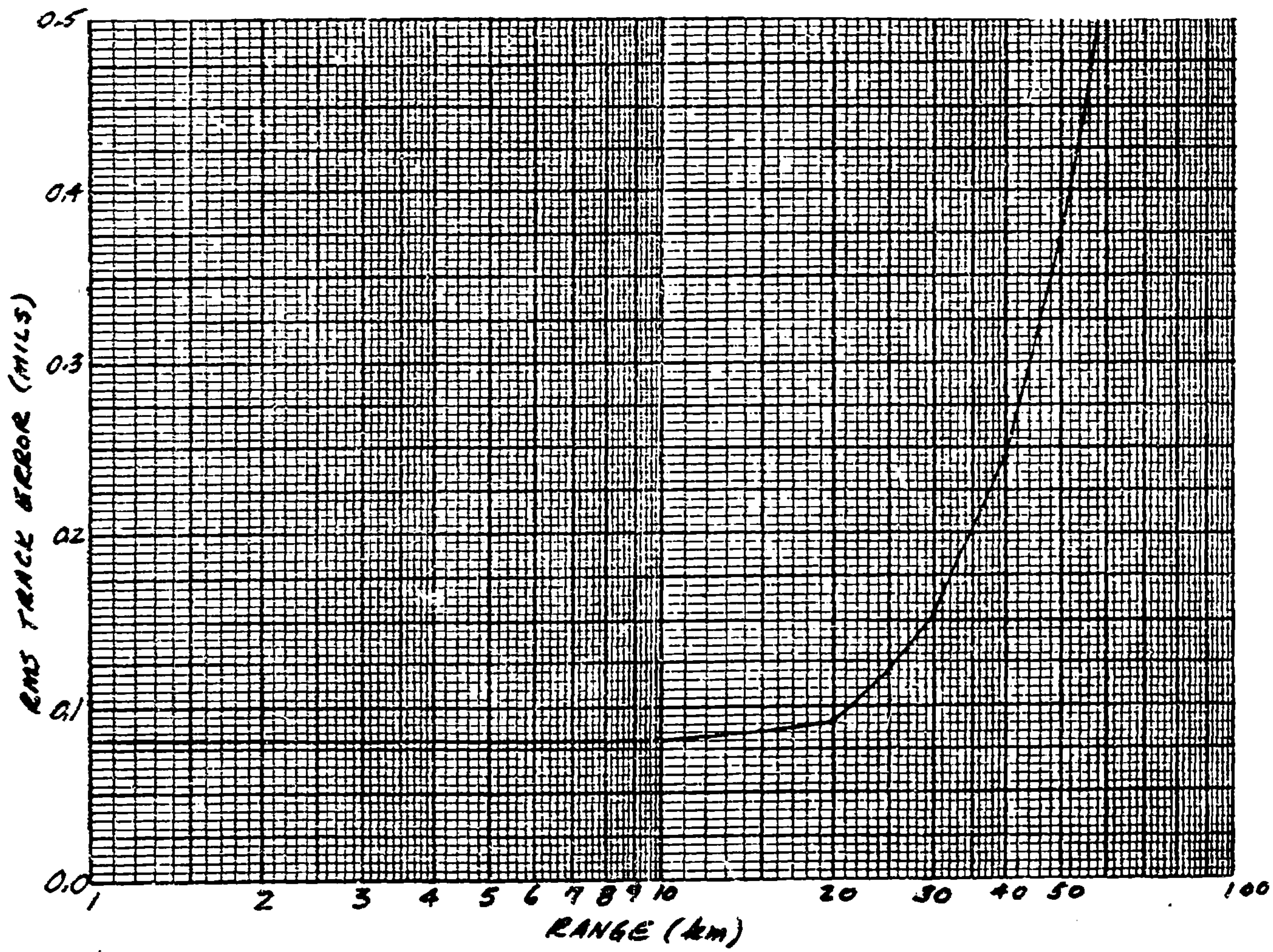


Figure 16. Tracking Errors as a Function of Range for the Limited Scan Phased Array Described in Table IV Tracking a Missile Equipped with a Passive Reflector

of cost and complexity. The major problem that is perceived at this time is the provision of polarization insensitive phase shifters to enable such limited scan concepts to be implemented in a dual polarized system.

III. BEACON/TRANSPONDER CONSIDERATIONS

A. Introduction

Preliminary design studies defining the generic characteristics of a radar system for the command guidance of a ballistic missile indirect fire system[3] indicated the inherent system advantages of including a beacon/transponder on board the missile. Some of the basic limitations of the passive reflector tracking radar approach to providing data on missile position and trajectory sufficiently accurate for command guidance which were identified in this previous study included low signal-to-noise ratios in a degraded weather (rain, fog, etc.) environment, clutter interference (ground clutter and rain), and depolarization of the returned signal due to returns from the missile body.

The use of an active beacon/transponder on-board the missile results in certain simplifications in the ground-based tracking radar and reduces the effect on overall system performance of the limitations discussed above. In particular, a missile transponder (offset in frequency from the radar transmitter) would:

(1) eliminate pulse-to-pulse integration due to large signal-to-noise ratios realized;

(2) eliminate the need for a coherent (Doppler) radar processing if the transponder frequency is offset from the transmitter frequency;

(3) remove the depolarizing effects of missile reflectivity from the signal received at the radar.

However, these advantages must be compared, in a cost effectiveness sense, with the increased cost and complexity of each missile. This cost comparison and performance effectiveness tradeoff will be the subject of the next two chapters.

B. Transponder Design Tradeoffs and Evaluation

Although a basic radar transponder is a relatively simple RF system, this particular application requires some unique operating characteristics and, thus, a careful examination of some of the transponder design questions will be undertaken.

1. Microwave Source

The single, most important performance and cost driver in the transponder is the source of microwave energy, its associated modulator, and power supply. The selected source must provide performance levels which match, or exceed, the general requirements for the missile beacon/transponder given in Table V. In addition and, perhaps, just as importantly, the source must be small, lightweight, producible in large quantities, and inexpensive. All of these requirements, plus the rapidly expanding technology, favor a solid-state source.

Several recent state-of-art comparisons among the various types of solid-state microwave source devices have recently appeared in the literature [21 - 26]. Information extracted from these references and other, more general, sources which compare the performance and characteristics of the three most promising solid-state source technologies--GUNN Diodes, IMPATTS, and LSA (Limited Space-Charge accumulation) Devices--for this application is given in Table VI. System characteristics most directly effecting overall

TABLE V

MISSILE BEACON/TRANSPONDER
PERFORMANCE PARAMETERS

Frequency	X-Band (9.5 GHz--Offset from Radar Frequency)
Delay Time	Fixed
Pulse Length	0.25 μsec
PRF	3750 Hz
Duty Factor	.001
Peak Power	$\geq$ 10 watts
Intrapulse Chirp	$\leq$ 4 MHz

TABLE VI

TRANSPONDER SOURCE SELECTION AND TRADEOFFS

	DEVICE		
CHARACTERISTIC	GUNN	IMPATT	LSA
Power	2	2	1
Efficiency	2	1	(2)
Stability	1	2	(2)
Cost	1	1	3
Availability	1	2	3
Spec Control	1	1	3
Reproducibility	1	1	3
Design Experience	1	2	3
Total:	10	12	20

Trades favor GUNN Technology at the present time performance for the missile transponder application have been selected for comparison. Most of the comparison made in this table are judgmental, relative and qualitative, rather than strictly quantitative. Relative comparisons were made among the devices and scores assigned in each category, with a score of 1 being the highest and 3 the lowest.

As indicated in this table, GUNN devices sources, primarily because of their relative advantages over IMPATT devices in the areas of availability and design experience, were slightly favored over IMPATT technology. Should the operating frequency have been higher, IMPATTS would probably have been favored. Also, technology trends will probably result in IMPATT being favored at some time in the future. LSA technology, even though initial experimentation held promise of very high power levels at microwave frequencies, has failed to meet expectations and has not, as yet, proven practical.

A review of available "off-the-shelf" GUNN and IMPATT sources has identified the Plessey-Type Series GDP010/001 has the source device which currently provides the closest match to the missile transponder requirements. The basic data on this device, as extracted from a current Plessey catalog, is given in Table VII. Note that the 10 watt device meets the frequency chirp requirements for this application.

2. Transponder Analysis and Parameter Evaluation

Some limited analyses of several transponder-equipped missile systems questions (such as received S/N and resulting tracking accuracy) were performed in the initial study.[3] However, no parametric trades and evaluation were considered at that time.

TABLE VII

X-BAND GUNN SOLID-STATE SOURCE CHARACTERISTICS

X BAND

Temperature compensated   Type GDP010/001

Frequency Range		9-11			GHz
Output Power	5	10	20	30	W peak
Operating Voltage	30-40	30-40	40-50	40-50	V
Operating Current	4	6	8	10	A
Rise Time	20	20	20	30	nS
Chirp	1	2	5	10	MHz
Pulse Width	0.5	0.5	0.5	0.5	μS
Duty Factor	1	1	0.5	0.1	%
Temperature Coefficient	±15	±15	±15	±15	kHz/°C
Temperature Range		-40 to +70			°C

For a transponder-equipped target, the two propagation paths associated with the radar range operation are effectively decoupled, such that the system evaluation breaks down into two, independent parts--the radar-to-beacon path and the beacon-to-radar path. Power levels received at the transponder and those returned to the radar are governed by the "beacon range equation" [27,28]. The signal available at the beacon receiver is given by

$$S_B = \frac{P_r G_r G_B \lambda^2}{(4\pi)^2 R^2 L_r L_B} \tag{11}$$

where

S_B = Power Received at Transponder

P_r = Radar Peak Power

G_r = Radar Antenna Gain

G_B = Beacon Antenna Gain

λ = Wavelength

R = Interrogation Range

L_r = Radar Losses

L_B = Beacon Losses

The ratio of signal received at the beacon receiver, S_B, and the minimum power required to trigger the beacon, S_{min}, (or the receiver sensitivity) is generally defined as the system gain margin for interrogation. Table VIII gives calculated gain margins for the following assumed system parameters:

TABLE VIII

RADAR/TRANSPONDER GAIN MARGIN

R(km)	SB(dBm)	Gain Margin Receiver Sensitivity -40 dBm	 -60 dBm
1	22.7	62.7	82.7
5	8.7	48.7	68.7
10	2.7	42.7	62.7
20	-3.3	36.7	56.7
40	-9.3	30.7	50.7
60	-12.9	27.1	47.1
100	-17.3	22.7	42.7

P_r = 250 KW

G_r = 41.5 dB

G_B = 19.2 dB

λ = 0.0316 meters (X-band)

L_r = 4 dB

L_B = 6 dB

The same, general expression as used for the signal available at the beacon applies for the signal level available at the radar receiver when P_r is replaced by P_B, the beacon power. Since noise power referred to the radar receiver, input is

$$N = K\, t_o\, B\, F_n \tag{12}$$

then, the resulting S/N received at the radar is given by

$$S/N = \frac{P_B G_r G_B \lambda^2}{(4\pi)^2 R^2 L_r L_B\, K\, t_o\, B\, F_n} \tag{13}$$

where

P_B = Beacon/Transponder Peak Power

K = Boltzmann's Constant

t_o = Temperature (Ambient)

B = Receiver Bandwidth

F_n = Receiver Noise Figure

For a radar receiver having a bandwidth of 4 MHz and noise figure of 9 dB, the received S/N as a function of range for several transponder peak power levels is given in Table IX.

Theoretical transponder tracking range performance for the set of system parameters previously assumed for these calculations and in terms of interrogation gain margin and received signal-to-noise ratio is given in Figure 17. Balance between the up-link to the transponder and the down-link to the radar is desirable in the overall system since the reply link must, at least, match the transmission path to obtain the desired range performance. Excess power in either link is wasted. The effective beacon sensitivity in parentheses corresponds to a radar peak power of 25 kw. The other set of receiver sensitivities corresponds to a radar peak power of 250 kw.

3. Transponder System Considerations

Some general conclusions regarding several of the missile transponders and radar system parameters can be developed from close examination of Figure 17. A relatively simple transponder crystal video receiver can provide a sensitivity of at least -45 dBm, and such a receiver operating in conjunction with a radar peak power of 25 kw will provide gain margins in excess of 18 dB to a maximum range of 100 km. Also, 10 watts of peak beacon power is sufficient to provide large radar signal-to-noise ratios (S/N $\geq$ 36 dB at 100 km) and, thereby, excellent tracking accuracies.

Note that over the expected missile operational range of 0-60 km, the gain margin varies by 40 dB. This large variation in gain margin implies that, for any reasonable transponder sensitivity, transponder interrogation

TABLE IX

RADAR SIGNAL-TO-NOISE RATIO

	Received S/N		
	Transponder Power		
Range (km)	1 Watt	10 Watts	100 Watts
1	67.7 dB	77.7 dB	87.7 dB
5	53.7	63.7	73.7
10	47.7	57.7	67.7
20	41.7	51.7	61.7
40	35.7	45.7	55.7
60	32.1	42.1	52.1
100	27.7	37.7	47.7

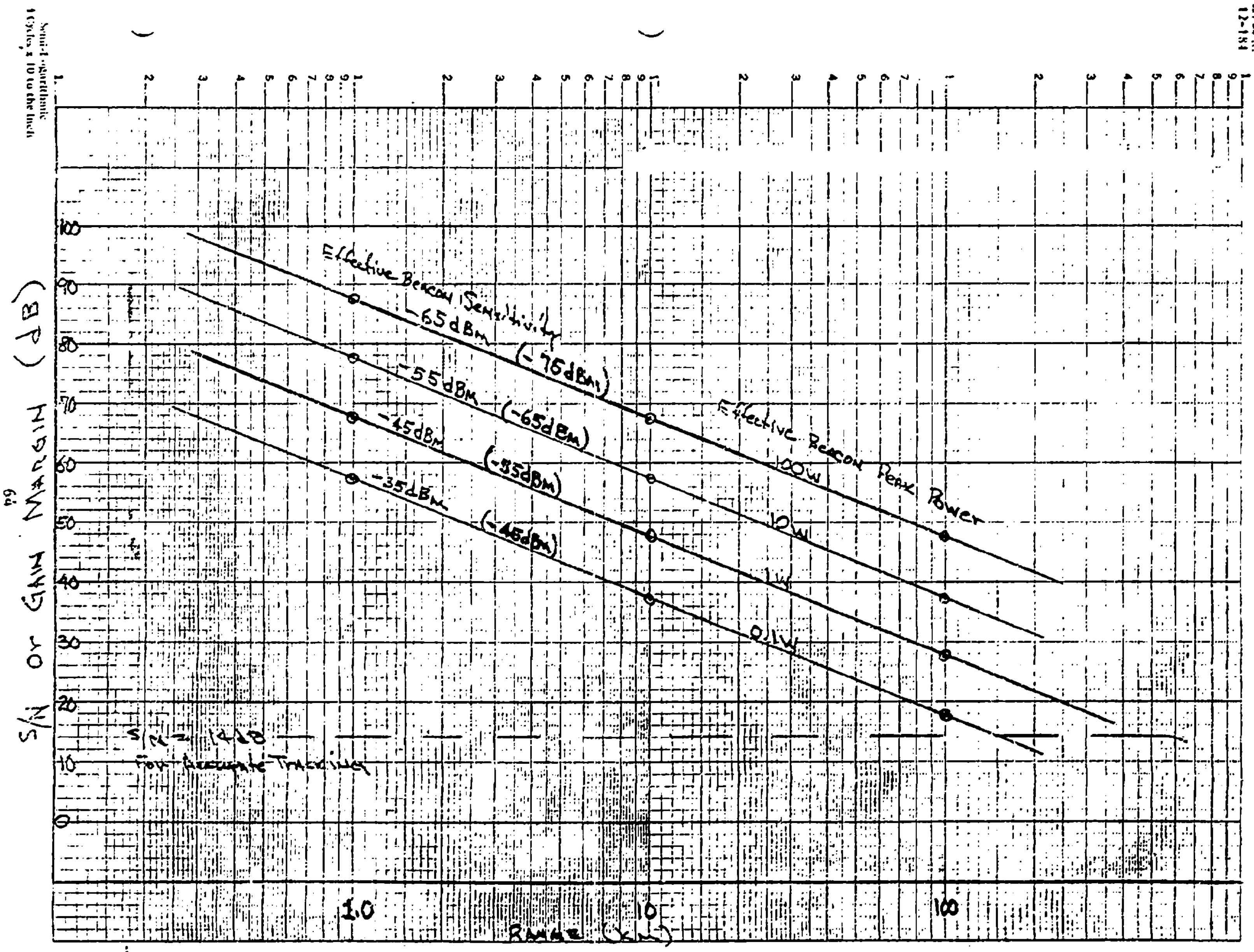


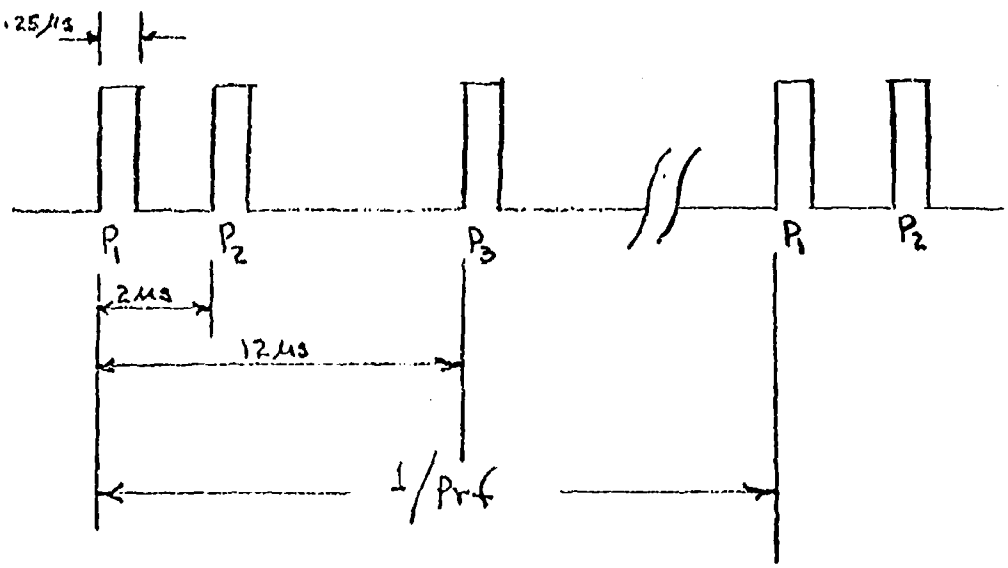
Figure 17. Theoretical Beacon Tracking Range for Missile Tracking Radar

commands radiated through the radar sidelobes would trigger the transponder. This may or may not represent a problem for the radar/transponder combination, since the missile may not be visible to another radar's sidelobes depending on the, as yet undefined, operational scenario. Also, sidelobe responses of the transponder may not confuse the tracking radar since all missile targets will be flying well-controlled trajectories and separated in range.

If sidelobe responses do result in significant, unwanted "clutter", then these responses can be eliminated by use of a sidelobe suppression circuit (SSC) in the transponder or, perhaps, by controlling the radar transmitted power as a function of range. The sidelobe suppression techniques require transmission of a second suppression pulse shortly after the main radar pulse. The suppression pulse is transmitted through a separate auxilliary "OMNI" antenna or through the monopulse antenna difference pattern. A more complete description of a sidelobe suppression implementation is given in reference.[27] One possible pulse coding and timing scheme is shown in Figure 18. P_1 represents the primary radar transmission pulse; P_2 the sidelobe suppression pulse; and P_3 the firing command pulse.

4. Transponder Block Diagram

A missile beacon/transponder design is shown in Figure 19 in block diagram form. A straightforward crystal video detection/video amplifier comparator combination is used in this transponder receiver. A pre-selection filter is used to establish the basic frequency selectability of the receiver and to prevent the transmitted signal from feeding back into the receiver. A sidelobe suppression circuit and a duty cycle overload circuit are included to prevent radar sidelobe interrogations and to limit the GUNN oscillator duty



P1 - PRIMARY Code Pulse

P2 - Sidelobe Suppression Pulse

P3 - Side Thruster Firing Command

Figure 18. Pulse Coding and Timing Diagram

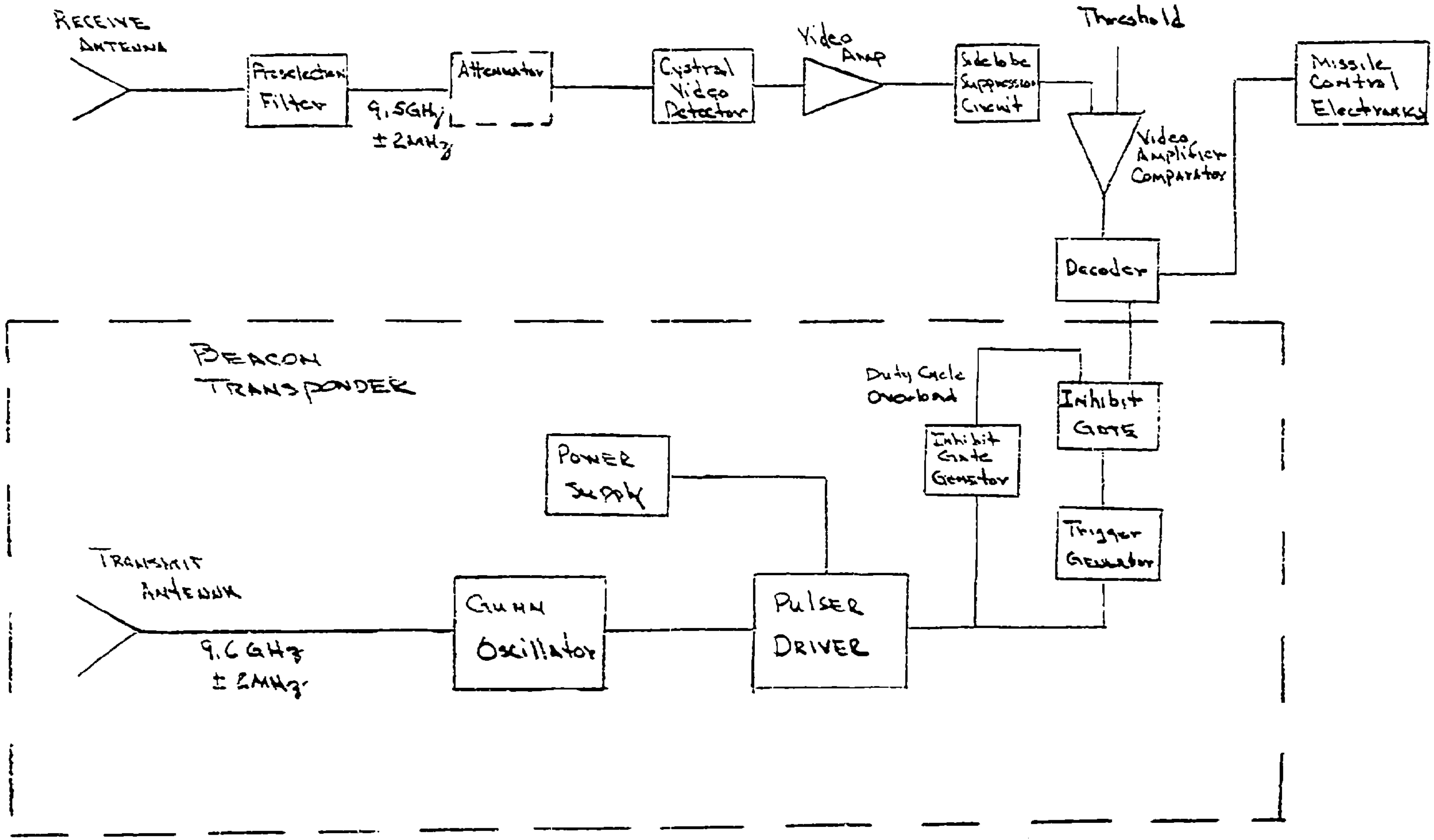


Figure 19. Missile Beacon/Transponder Block Diagram

factor to the specified value. Separate receiving and transmitting antennas are also used.

Since the basic GROWLAR missile will always have an on-board receiver, regardless of whether a passive reflector or transponder-equipped missile concept is employed, the only extra cost items to be included in the missile electronics are those components enclosed within the dashed-box and the side-lobe suppression circuit, if one is required.

IV. TRANSPONDER COST ESTIMATES

One of the most important factors associated with the evaluation of the transponder-equipped missile approach is the cost added to the individual missile due to the transponder. In an effort to quantify transponder cost, Plessey Microsystems was contacted for detailed cost estimates for large-quantity buys of GUNN microwave power sources.[29] In quantities of 10,000 units, or larger, and projecting a continuing development of GUNN fabrication technology over the next two to three years, the Plessey 10-watt, X-band GUNN diode source, GDP010-001 is expected to cost less than $100 per unit.

A power supply design previously developed at Georgia Tech for a similar Plessey C-band GUNN diode source is shown in Figure 20.[30] This design includes 3 transistors, 6 diodes, an SCR, and a DC-to-DC converter. Again, in production quantities of more than 10,000 units, the production and fabrication cost of this part of the transponder design is estimated not to exceed $40 per unit.

The pulser/driver design for the same C-band transponder included 5 active transistors and various passive circuit components. The production cost for this circuit is estimated to be less than $20/unit for quantities of more than 10,000 where medium-scale integration techniques could be used for both this circuit and the logic circuits required for the duty cycle overload circuit, inhibit gate, and trigger generator. Also, in large production runs, the antennas used in this transponder could be very simple and inexpensive stamped horn designs.

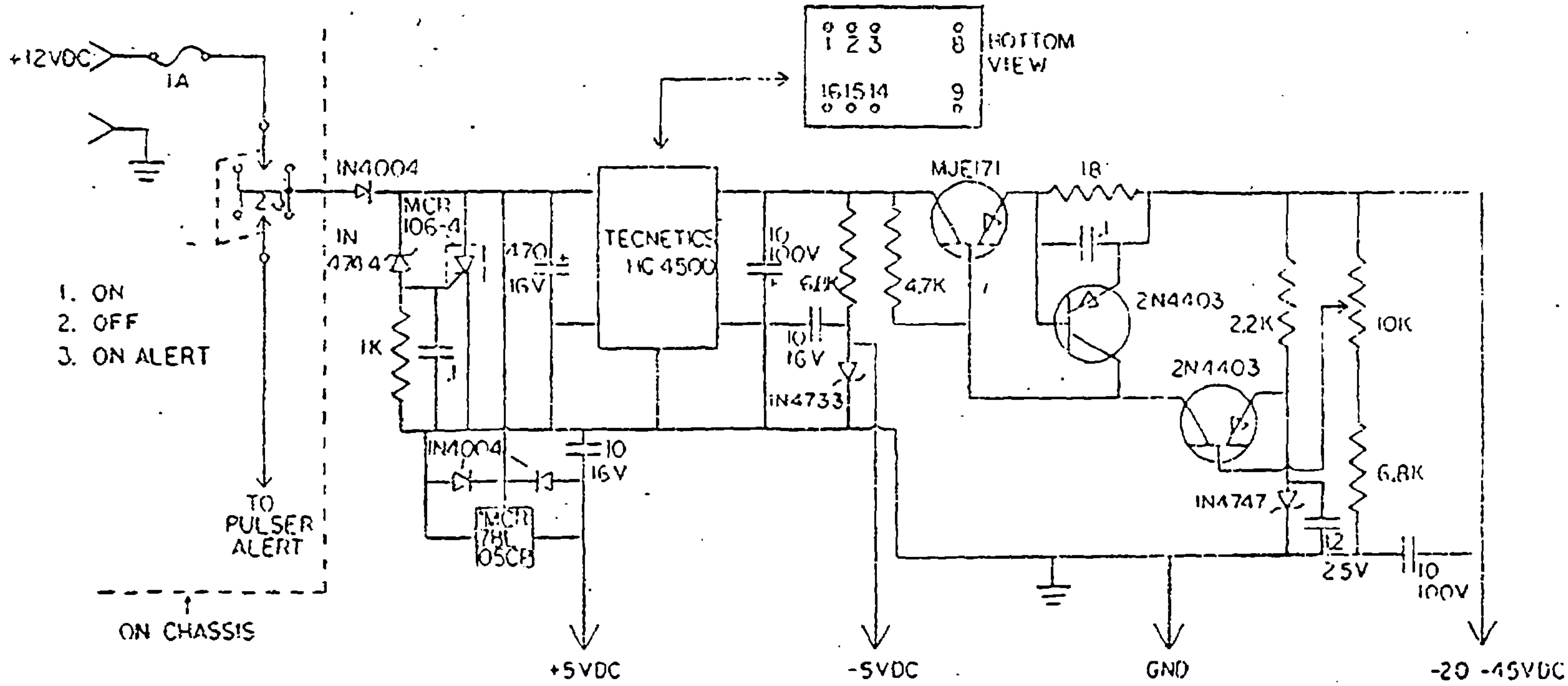


Figure 20. Typical Power Supply for GUNN Diode Source

Based on these considerations, a reasonable estimate of production costs associated with the fabrication and assembly of the beacon/transponder components shown in Figure 19 are given below:

Component	Estimated Cost
Oscillator--Plessey GDP010 (Quantity: $\geq$ 10,000)	\$100/each
Power Supply	40
Pulser/Driver	20
Logic	10
Antenna	10
Total Per Unit	\$180

This cost estimate closely agrees with a "rule-of-thumb" transponder cost estimating technique used by Plessey.[29] Previous experience of Plessey engineers has indicated that the source modulator and power supply cost approximately equal the cost of the basic source device. Applying Plessey's rule-of-thumb to this particular transponder design requirement results in a cost estimate of \$200/unit versus \$180/unit when individual subcomponent costs are estimated.

V. MISSILE RADAR CROSS SECTION AUGMENTATION

One of the difficulties encountered when utilizing a radar for tracking a missile equipped with only a passive reflector is that the radar cross section of a typical missile is relatively small--particularly when viewed from a rear aspect. In addition, if polarization measurement is desired in order to determine the roll angle of the missile, the polarization of the backscattered energy must be controlled with reasonable precision. A rather comprehensive, general discussion of the broad area of the radar cross section augmentation of missiles is included in Hosking,[31] so this discussion will be limited to factors specifically applicable to the GROWLAR concept. Two general classes of missile reflectors will be considered: those which are fin-mounted, and those which could be attached to--or imbedded in--the body of the missile.

A. Fin-Mounted Reflectors

In the GROWLAR concept, the missile will be viewed by a radar from a predominantly rear aspect. For the most extreme trajectory which is likely to be encountered, aspect angles from 0 to from 30-40° off a rear axis of the missile will normally be experienced. One location for a reflector to enhance radar cross section is on the trailing edge of the missile fins. Earlier discussions have indicated that fins as large as 5½ inches long by 2 inches wide may be accommodated on a GROWLAR missile without adversely affecting its aerodynamic properties.

Analyses set forth in the earlier report indicated that such an area, if completely utilized, could provide significant radar cross section enhancement. However, only maximum values were considered in this earlier analysis, and in order to define the properties of such a fin-mounted reflector more fully, additional investigations have been carried out. The specific configuration which was analyzed was a Van Atta array consisting of six waveguide horns (three pairs of connected horns). Such a configuration is quite amenable to inexpensive fabrication using conventional sheet metal stamping and forming techniques on a mass production basis. In addition, such a concept has the advantage of having a high degree of polarization purity so long as the connecting waveguide and horns propagate only the dominant mode.

The antenna pattern of such an array is determined, primarily, by the pattern of the individual elements which constitute the array. Figure 21 gives the relationship between the aperture dimensions and the 3 and 10 dB beamwidths of the radiation pattern of a rectangular waveguide, which should be quite similar to that for the horns required for the array since only a small flare is required. Examination of Figure 21 shows that for a 3-dB beamwidth of 60°, an aperture of approximately one wavelength in the H plane and an aperture of approximately 1¼ wavelength in the E plane would be required. For operation near X-band, where wavelengths are approximately one inch (3cm), such a configuration fits well into the available fin.

The utilization of a non-ideal antenna elements having an inefficient utilization of the aperture (due to illuminative taper and reflections) results in a decrease of effective radar cross section over the maximum values which were calculated earlier. The reduction in area is approximately 2%, representing the difference of the width of the horn of approximately

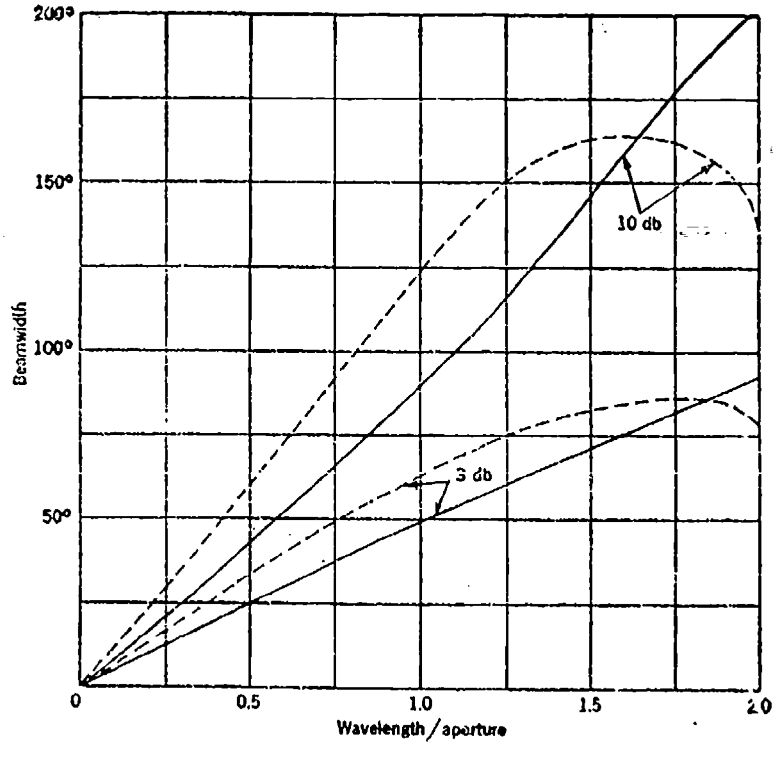


Figure 21. Relationship Between Aperture Dimension and the 3 dB and 10 dB Widths of the Radiation Pattern of Rectangular Waveguide: ———E-Plane; ----- H-Plane[32]

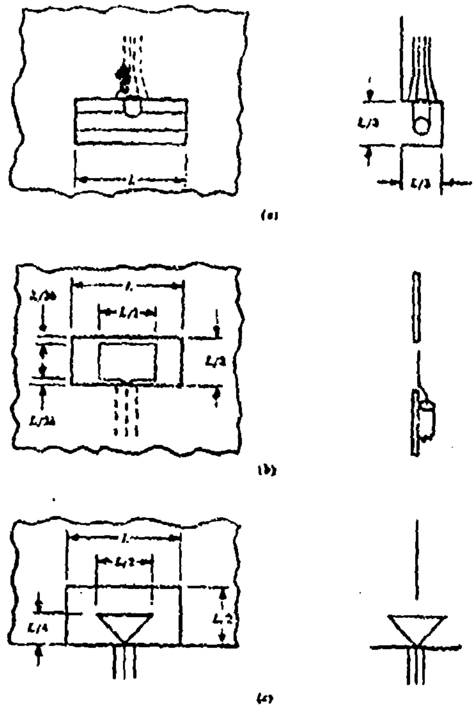
4 centimeters and the approximate 5 centimeter maximum width which could be utilized within the fin (approximately 1 dB). Another gain reduction is associated with the inefficient utilization of the available aperture. The gain achievable from such an open end waveguide feed is given by approximately 10.2 times the physical area divided by the square of the operating wavelength. This is contrasted with the maximum gain which could be achieved, which would have 4π rather than 10.2 as the multiplicative factor. Thus, there is an additional gain reduction of approximately $10.2/4\pi$, or 1 dB.

Thus, the achievable cross section given by a Van Atta array installed in a single fin of the missile is approximately 2 dB less than the maximum achievable values calculated in the earlier report.

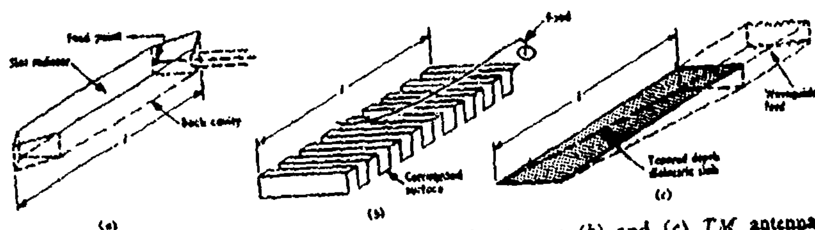
B. Body-Mounted Reflectors

There would be some advantage obtained if the reflectors could be mounted on the body of the missile itself--preferably on a removable section of the missile--thus requiring no re-design of the missile fins. There are two major difficulties which are associated with such a concept: first, for aerodynamic reasons, any protrusions from the surface must be relatively small, yet the available area to intercept incident electromagnetic energy must be large when viewed in the direction of the radar; second, the polarization must be carefully controlled completely around the missile, over a relatively large viewing angle.

While there have been a number of flush-mounted antenna/reflector configurations which are based on either dielectric or slot antennas,[33,34] as shown in Figure 22, or which utilize Luneburg lenses behind a radome, none of these



Slot antennas, front views and cross sections.



Surface-wave antennas. (a) *TE* antenna; (b) and (c) *TM* antennas.

Figure 22. Slot and Surface Wave Antennas Which May Also be Used for Body-Mounted Reflectors [33]

configurations are suitable for radiation directly back to the rear of the missile, nor are they capable of easily producing other than a single polarization relative to the location of the ground plane. That is, different reflectors around the missile would produce different polarizations, thus rendering the use of polarization measurement for roll angle determination ineffective.

Some consideration was given to the concept of cross section augmentation for only one roll angle position, thus producing a significant increase in backscattered energy as that portion of the missile came into view of the radar. The problems which would exist with such a system include achieving appreciable augmentation near the tail aspect, and radar tracking problems associated with the significantly reduced data rate. Consideration of these problems leads to the conclusion that such a concept would involve a relatively large, complex, and expensive radar system, having significantly reduced performance.

VI. MULTIPLE MISSILE TRACKING RADAR COST ANALYSIS

Two, viable, multiple missile tracking radar concepts emerged from the analysis and system evaluation given in Chapter II of this report: (1) a limited scan phased array concept tracking a passive reflector-equipped missile, and (2) an off-axis tracking monopulse concept working in conjunction with a beacon-equipped missile. The costs associated with both of these concepts will be analyzed in this chapter and a cost comparison/tradeoff among the free rocket, the basic GROWLAR concept, a limited scan radar, and the off-axis monopulse system will be developed and presented.

A. Limited Scan Phased Array

Overall system cost benefit and performance improvement resulting from the incorporation of command guidance into the Army's General Support Rocket will be a major evaluation criterion in determining the viability of the command guidance concept in comparison with a free-flight ballistic rocket with no guidance. For that reason, reliable and accurate cost projection for each of the candidate radar tracking techniques must be generated.

Estimating costs of complex military systems such as radars is an extremely difficult challenge in this time of rapidly escalating costs (inflation) coupled with extremely rapid technology advances in the areas of digital and solid state electronics which tend to offset inflation factors.
To prepare a believable cost estimate under such conditions requires a large cost data base, coupled with extensive experience and access to a validated military systems cost model. Previous efforts at Georgia Tech[35] have provided all three of these. However, a major pitfall in cost estimation of

military equipment derives from the attempt to forecast or project future costs based on current or previous information. As previously indicated, neither rapid technological change, state-of-the-art advances, nor the precise specification of development events which require creativity and innovation are known *a priori*. Thus, a conclusion by many analysts[36] is that absolute future costs can never be established.

Using cost estimating relationships developed and described in reference,[35] drawing on an extensive cost data base for similar coherent radar system either currently under development in production (i.e. AN/TPQ-36, AN/TPQ-37, AN/PPS-5, etc.) and obtaining additional new and current cost data on relevant radar systems through direct, personal contact,[37] the unit production costs, in quantities of 106 units, for a limited scan missile tracking phased array radar having the operational parameters identified in reference in reference 3 and further refined Chapter II of this study, were established. The results of the cost estimation exercise is shown in Table X. Cost estimates for the major radar sub-components are given in this Table.

The primary cost drivers for this radar are the rather sophisticated limited scan, dual-polarized, monopulse tracking antenna, the coherent digital MTI, CFAR, and tracking processor, and the coherent 250 kw peak power, pulse compression transmitter. The final assembly and test cost is based on a validated algorithm which relates the basic complexity of the total system integration and the level of the total subsystems costs through learning curve functions (determined by the number of production units) to final assembly and test costs.

For a production run of 106 units, the estimated unit production cost for the limited scan phased array concept is $546,400 in 1978 dollars.

TABLE X

LIMITED SCAN PHASED ARRAY RADAR COST ESTIMATE

Component	Cost
Transmitter (Coherent, MOPA, 250 kw, Pulse Compression)	$59K
Antenna (Limited Scan, Single Plane, Phased Array, Dual Polarized, Monopulse)	128K
Receiver (Monopulse)	30K
Processor (Digital, MTI, Coherent, Tracking, CFAR)	130K
Display (A-Scope, Maintenance)	5K
Shelter	10K
Prime Power	5K
Final Assembly and Test	179.4K
TOTAL	$546.4K/each

B. Off-Axis Tracking Monopulse

Using the general design parameters for the off-axis tracking monopulse radar concept presented in Chapter II and the same basic radar cost data base (which included several non-coherent radar systems) and cost estimating relationships as were used for estimating the costs of the limited scan system, a cost estimate was developed for the off-axis monopulse. This estimation is detailed in Table XI.

Again, the major cost drivers were the antenna and the processor; however, since both of these sub-components have been considerably reduced in complexity when compared with their counterparts for the limited scan radar, the individual sub-component costs are lower. Also, the transmitter and receiver are less costly, primarily since both are non-coherent and the transmitter has a lower peak power for this radar concept.

For a production run of 106 units, the estimated unit production cost for the off-axis monopulse is $200,000 in 1978 dollars.

C. Relative Cost Effectiveness of Missile Systems

A relative cost effectiveness comparison between the two missile tracking radar concepts identified herein (the limited scan and off-axis tracking radars) the original GROWLAR concept, and the unmodified, totally ballistic or free-flight rocket is presented in Table XII. Data on missile costs, including the propulsion unit, warhead, basic electronics, and thrusters--together with cost data for the basic GROWLAR tracker--were supplied by MIRADCOM for this cost comparison. In all cases, tracker life cycle costs were assumed to be 100 percent of hardware costs, and tracker unit costs were pro-rated over the number of missiles for a requirement of 106 trackers.

An extremely important conclusion is immediately obvious from this table, even though the off-axis monopulse tracking concept requires more electronics

TABLE XI

OFF-AXIS TRACKING MONOPULSE RADAR COST ESTIMATE

Component	Cost
Transmitter (Non-Coherent, 25 kw, Frequency Tuned)	$10K
Antenna (Reflector or Slotted Array, Dual Polarized, Monopulse, Non-Scanning, Pedestal)	40K
Receiver (Non-Coherent, Monopulse)	20K
Processor (Digital, Tracking, Amplitude Threshold)	45K
Display (A-Scope, Maintenance)	5K
Shelter	10K
Prime Power	5K
Final Assembly and Test	65K
TOTAL	$200K/each

TABLE XII

MISSILE COST EFFECTIVENESS

	Free Rocket	GROWLAR	Radar-LS	Radar-OA
Rounds	150,000	112,000	108,000	117,000
Cost Ratio	1.00	1.35	1.39	1.28
Costs				
Propulsion Unit	$1,422	$1,537	$1,537	$1,537
Warhead	2,339	2,229	2,229	2,229
Missile Add-On		84	84	84
Electronics		320	320	320
Thrusters		458	537	181
Tracker (Pro-Rated)		458	537	181
Tracker Life Cycle				180
Transponder				100
Transponder Life Cycle				
TOTALS:	$3,761	$5,086	$5,244	$4,812

on-board the missile (transponder), this concept is more cost effective than the limited scan concept by a significant margin due to the simplified radar tracker requirement and the resulting reduction in tracker cost pro-rated for each missile.

VII. CONCLUSIONS AND RECOMMENDATIONS

This report and its predecessor, "Preliminary Design Study for a Command-Guided Ballistic Missile Radar",[3] have addressed in detail many of the technical questions associated with developing and implementing a command-guided ballistic rocket using a microwave or millimeter radar to provide position and roll information for the rocket. Using extensive design calculations, performance trade-offs and projections, and supporting technical analysis, the basic feasibility of an X-band radar for providing the rocket position and roll data was established in the first phase of this design study.

This second phase of the investigation concentrated on providing additional depth in the treatment of the missile retroreflector configurations, more detailed performance and cost trade-offs among the beacon and passive reflector-equipped missile concepts, missile beacon transponder design trade-offs and a recommended design configuration, and an analytical examination of multiple rocket tracking system concepts and tracking radar requirements. The primary conclusions evolving from these continuing studies and analyses described in this report are summarized below:

(1) Multiple missile tracking of a transponder-augmented missile may be accomplished with a low-peak power, relatively low cost, off-axis tracking conventional monopulse radar system operating at X-band.

(2) Use of passive retroflectors on-board the missile requires a more complex and expensive ground-based radar tracking system.

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(3) Solid state missile transponders suitable for this application may be developed and built at low cost in large quantities.

(4) Passive missile RCS augmentation appears promising only when the retroreflector is mounted on, or in the trailing edge of, a missile fin.

(5) A transponder-equipped missile in conjunction with a monopulse tracking radar appears to be the most desirable approach, due to lower overall system cost and increased tracking accuracy.

(6) Development of a "Proof-of-Concept" command-guided ballistic missile radar system based on the design recommendations developed in this study appears feasible at this time and is recommended.

During the course of these studies, several areas requiring additional studies and investigations have become apparent:

(a) Radar-to-radar interference and transponder operation in a multiple radar environment (Electromagnetic Compatibility-EMC) and a realistic, electronic battlefield scenario.

(b) Tracking radar and overall system vulnerability to Electronic Countermeasures--ECM.

(c) More optimum track smoothing algorithms and procedures.

These additional technical questions requiring further analytical studies should be addressed but their investigation should not preclude, nor delay, initiation of a "Proof-of-Concept" system demonstration as outlined in the preceeding chapter.

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APPENDIX

BALLISTIC MISSILE TRACKING RADAR ANGULAR RATE ERRORS

For the ballistic missile tracking radar, consider the geometry shown in Figure A-1 with the radar and launcher co-located at the coordinate origin and with the trajectory confined to the vertical plane of launch. The basic objective of this analysis is to develop a set of expressions relating the errors in estimating $\dot{\phi}$ (t), the angular rate error, to the basic radar measurement errors--σ_R (the range error), $\sigma_{\dot{R}}$ (the range rate error), and σ_ϕ (the angular error). The following assumptions will hold throughout this analysis:

(1) $R(t)$ = Radar-measured range having Gaussian distributed errors with zero mean and a standard deviation, σ_R.

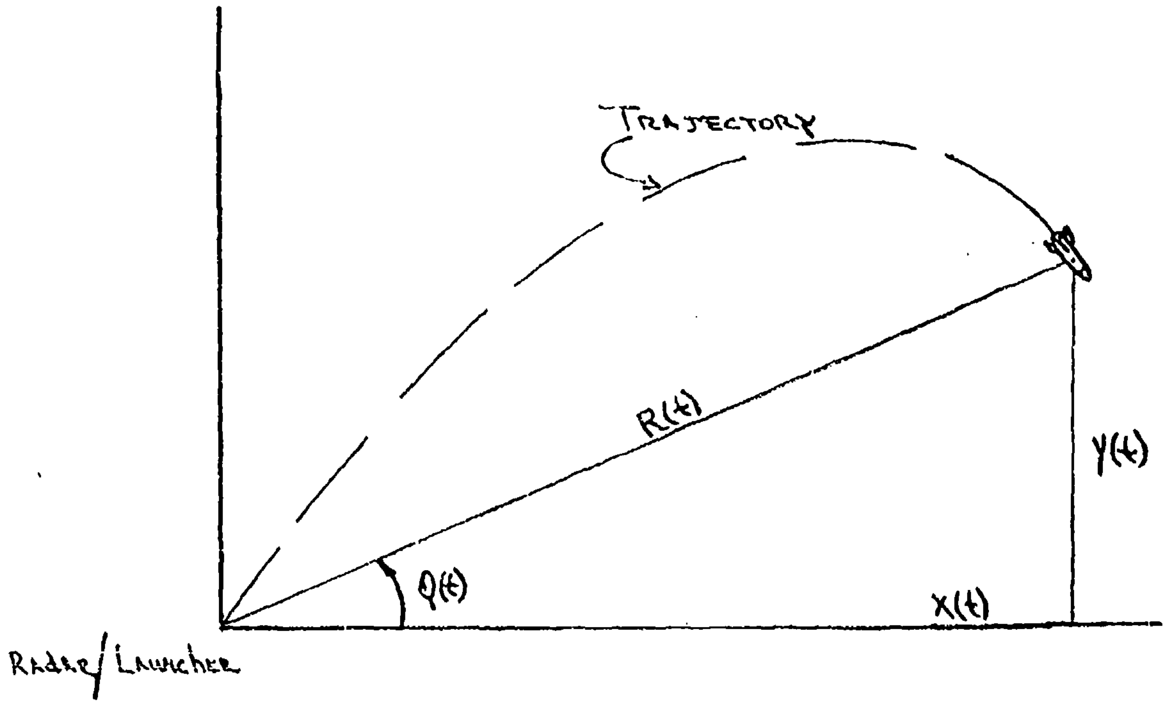
(2) $\phi(t)$ = Radar-measured elevation angle having Gaussian distributed errors with zero mean and a standard deviation, σ_ϕ.

(3) $X(t)$, $Y(t)$ = Cartesian coordinates of rocket at any time, t; X(t) and Y(t) are defined by the ballistic equations of motion.

Only the basic analysis steps and major analytical relationships will be outlined in this discussion. From the geometry of the problem, as shown in Figure A-1,

$$\text{Cos}\,\phi(t) = X(t)/R(t) \tag{A-1}$$

GEOMETRY



- RADAR/LAUNCHER COLOCATED
- TRAJECTORY CONFINED TO VERTICAL LAUNCH PLANE

Figure A-1. Radar-Missile Trajectory Geometry and, therefore, by differentiating in time,

$$\dot{\phi}(t) = \left(\frac{1}{R(t)\sin\phi(t)}\right)\left(\dot{R}(t)\cos\phi(t) - \dot{X}(t)\right) \tag{A-2}$$

Equation A-2 relates the angular rate, $\dot{\phi}(t)$, to the range to the target, the elevation angle, and the range rate; i.e.,

$$\dot{\phi} = f(R,\phi,\dot{R}) \tag{A-3}$$

Therefore, the total error in estimating $\dot{\phi}(t)$, $\delta\dot{\phi}$, from the radar measurements of range, elevation angle, and range rate can be expressed as:

$$\delta\dot{\phi} = \partial\dot{\phi}/_{\partial R}\ \delta R + \partial\dot{\phi}/_{\partial\phi}\ \delta\phi + \partial\dot{\phi}/_{\partial\dot{R}}\ \delta\dot{R} \tag{A-4}$$

where δR, $\delta\phi$, and $\delta\dot{R}$ are the radar measurement errors. Therefore, expressions for $\partial\dot{\phi}/\partial R$, $\partial\dot{\phi}/\partial\phi$, and $\partial\dot{\phi}/\partial\dot{R}$ must be derived starting from the basic relationship given in Equation A-2.

After taking the indicated derivatives and performing extensive algebraic manipulations,

$$\partial\dot{\phi}/_{\partial R} = \frac{\dot{X}-\dot{R}\cos\phi}{R^2\sin\phi} \tag{A-5}$$

$$\partial\dot{\phi}/\partial\phi = \frac{1}{R\sin^2\phi}\left[\dot{X}\cos\phi - \dot{R}\right] \qquad \text{(A-6)}$$

$$\partial\dot{\phi}/\partial\dot{R} = \frac{\cos\phi}{R\sin\phi} \qquad \text{(A-7)}$$

Now, assume that the radar measurement errors are independent, random variables, then Equation A-4 can be re-expressed in terms of the rms error in estimating $\dot{\phi}(t)$--$\sigma_{\dot{\phi}}$.

$$\sigma_{\dot{\phi}}^2 = \left(\partial\dot{\phi}/\partial R\right)^2 \sigma_R^2 + \left(\partial\dot{\phi}/\partial\phi\right)^2 \sigma_\phi^2$$

$$+ \left(\partial\dot{\phi}/\partial\dot{R}\right)^2 \sigma_{\dot{R}}^2 \qquad \text{(A-8)}$$

After substitution of the expressions given in Equations A-5 thru A-7 and re-arranging terms, $\sigma\dot{\phi}$ can be expressed in terms of the basic radar measurement errors.

$$\sigma_{\dot{\phi}}^{2} = \left(\frac{\sigma_R}{R\sin\phi}\right)^2 \left[\left(\frac{\dot{X}-\dot{R}\cos\phi}{R}\right)^2 + \left(\frac{\dot{Y}\cos\phi-\dot{R}}{\sin\phi}\right)^2 \left(\sigma_\phi/\sigma_R\right)^2\right.$$

$$\left. + \cos^2\phi \left(\sigma_{\dot{R}}/\sigma_R\right)^2\right] \qquad \text{(A-9)}$$

It can be shown that the first two terms inside the brackets of Equation A-9 are small in comparison with the last term; therefore,

$$\sigma_{\dot{\phi}}^{2} = \left(\frac{\sigma_R}{R\sin\phi}\right)^2 \left(\phi \left(\sigma_{\dot{R}}/\sigma_R\right)^2 \right) \qquad \text{(A-10)}$$

and finally,

$$\sigma_{\dot{\phi}} = \frac{\sigma_{\dot{R}}}{R\tan\phi} \qquad \text{(A-11)}$$

For the radar parameters used for the X-band tracking radar,

$$\sigma_{\phi} = 0.1$$

$$\sigma_R = 4.2 \text{ meters}$$

$$\sigma_{\dot{R}} = 0.39 \text{ m/sec}$$

a signal-to-noise ratio of 16 dB, and the following geometric conditions,

$$R = 30 \text{ km}$$

$$QE = 30^{o}$$

$$V_o = 1220 \text{ m/sec (muzzle velocity)}$$

the elevation angular rate errors can be calculated

$$\sigma_{\dot{\phi}} = 0.03 \text{ m rad/sec}$$

Under the same radar error parameters, the range-rate and angular-rate errors can be plotted versus range, as shown in Figure A-2.

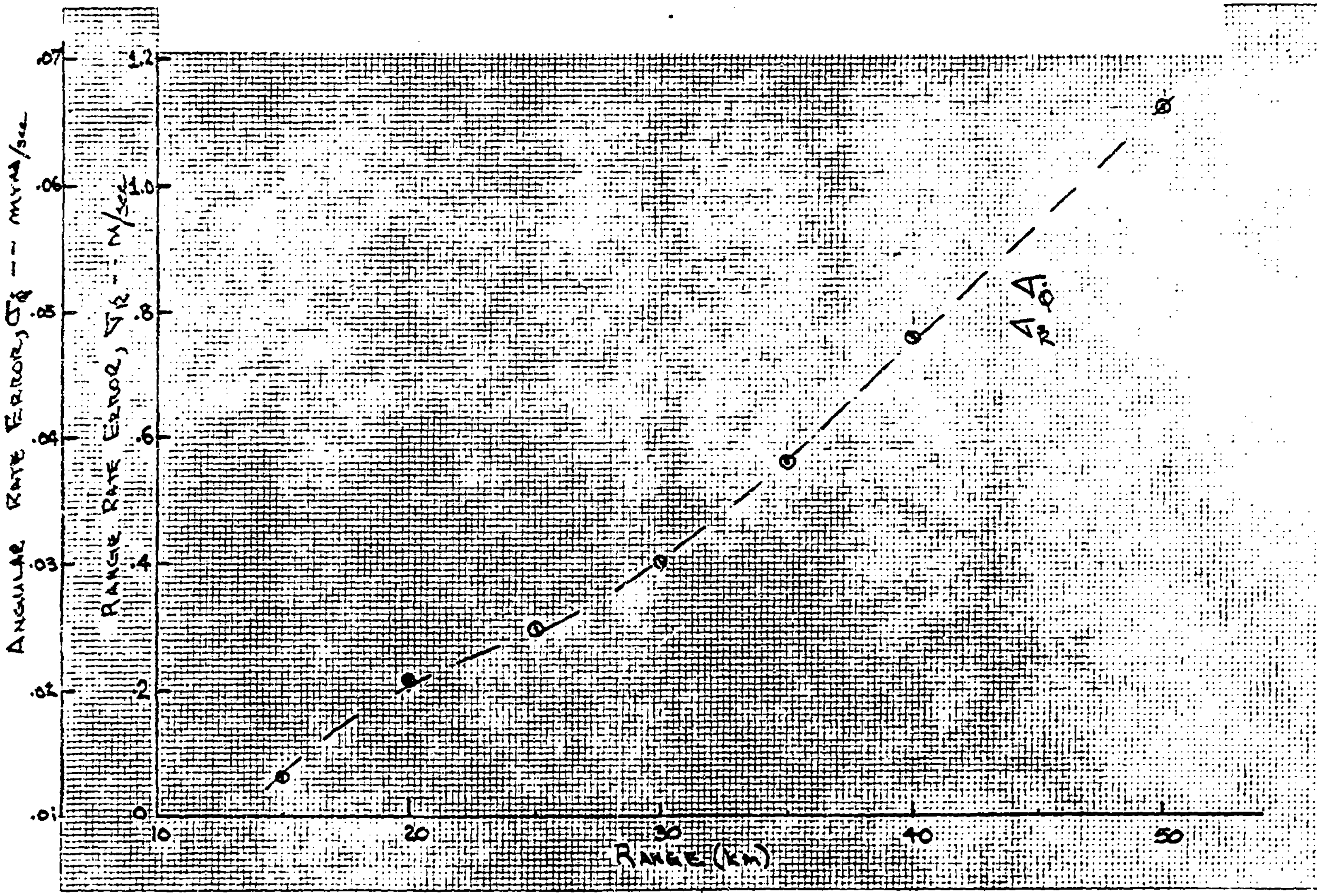


Figure A-2. Range and Angular Rate Errors vs Range

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